Returns to Scale without Functional Form: Identification, Falsification, and Evidence from U.S. Manufacturing

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Abstract

When do production choices support a common returns-to-scale parameter? We characterize the complete set of homogeneity degrees consistent with observed inputs, outputs, and input prices under cost minimization, without specifying a production function. The characterization yields sharp, closed-form bounds, rejection certificates involving at most two ordered comparisons, and a tractable index measuring the proportional relaxation of the revealed-cost inequalities. Applied to U.S. manufacturing industries, the framework rejects the joint hypothesis of a time-invariant homogeneous technology, exact cost minimization, and correctly measured data in every full industry panel. Exact consistency also disappears rapidly as consecutive observations are added, although departures can remain small over short horizons. Larger rationalizable subsets typically combine years separated by decades, and equally large subsets can imply different returns-to-scale classifications. Selecting observations can therefore deliver a precise classification without establishing a period of technological stability. The framework distinguishes the scale parameters supported by a maintained production model from the empirical adequacy of that model, making both model failure and sensitivity to sample selection explicit.

Keywords: Returns to scale, revealed preference, nonparametric production analysis, cost minimization, homogeneous production functions, partial identification.

JEL Codes: D24, C14, L11, D22, O33.

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1 Introduction

Returns to scale describe how output responds when all inputs expand proportionally. Empirical work often summarizes this relationship by a single parameter, even when the observations used to recover it span many years. Such a summary raises a prior question: can the observed production choices be reconciled with one homogeneous technology at all? If they cannot, choosing a more flexible functional form need not resolve the problem. The difficulty may lie in imposing a common technology on observations that cannot jointly satisfy its economic restrictions.

This paper develops and applies a nonparametric diagnostic for that question. Given observed inputs, outputs, and input prices, we ask whether one time-invariant production function, homogeneous of some degree $k > 0$, can rationalize all observations as cost-minimizing choices. The degree is unknown, and neither a parametric production function nor a parametric specification of substitution elasticities is imposed. When the data are rationalizable, the procedure recovers the complete set of admissible degrees. When they are not, it identifies the comparisons responsible for rejection and measures the relaxation needed to restore consistency. The same framework therefore connects identification, falsification, and the magnitude of model failure.

The distinction matters for interpreting production estimates. Output elasticities enter widely used measures of productivity and market power (Hall, 1988; De Loecker and Warzynski, 2012; De Loecker et al., 2020), while identification and specification choices can materially affect what those elasticities recover (Bond et al., 2021; Gandhi et al., 2020). Our exercise examines a specific maintained hypothesis behind a stable homogeneous representation: a common technology, exact cost minimization, and correctly measured quantities and prices. Rejection concerns this joint hypothesis. It does not separately establish technology change, non-optimizing behavior, or measurement error. It does, however, establish that changing the parametric functional form within the maintained homogeneous class cannot reconcile the observations.

The theoretical starting point is a power transformation. A production function f is homogeneous of degree k if and only if $f^{1/k}$ is homogeneous of degree one. Transforming observed output accordingly allows us to apply Varian's (1984) constant-returns cost-minimization test at any candidate degree (Theorem 1). This fixed-degree characterization is an application of Varian's result. The paper develops its implications for an unknown common degree and uses them to assess stability across production observations.

The transformation makes identification particularly transparent. Each ordered pair of observations restricts the admissible degree to an interval, possibly empty or unbounded.

Intersecting these restrictions yields the sharp identified set $\mathcal{K}(S)$ in closed form (Proposition 2). If the intersection is nonempty, every degree it contains admits a rationalizing technology; none outside it does. The resulting interval describes exactly which returns-to-scale conclusions the data support under the maintained hypothesis, without selecting a production function.

The interval structure also makes rejection interpretable. Every failure can be certified by at most two ordered pairs (Theorem 2). One comparison may be incompatible with every positive degree, or two comparisons may impose mutually inconsistent bounds on the common degree. These certificates distinguish an individually impossible comparison from a failure of compatibility across observations. A short certificate does not imply that rejection is easy to repair: many such conflicts can coexist within the same dataset.

To assess the extent of failure, we study two complementary measures. The cost-efficiency index $e^*(S;\mathbb{K})$ records the largest common multiplicative efficiency level consistent with the pairwise restrictions for some degree in a specified compact admissible interval $\mathbb{K}$. Its logarithm is the minimum of affine functions of $1/k$, so its computation reduces to a one-dimensional concave maximization problem. The index equals one exactly when a degree in $\mathbb{K}$ rationalizes the data. The associated slack, $1 - e^*$, measures departure from the restrictions; it is not an estimate of resources literally wasted in production. The deletion distance instead asks how many observations must be removed to obtain an exactly rationalizable dataset. Together, the two measures distinguish relaxing cost comparisons from reducing the set of observations the model must explain.

We apply the framework separately to 361 U.S. manufacturing industries in the NBER-CES database over 1958–2016. Every industry fails the exact test over the full panel, and removing a single year never restores rationalizability. The central empirical finding, however, is how rapidly consistency disappears as the observation horizon expands. Approximately 62% of adjacent two-year windows pass, compared with only 0.5% of three-year windows. In the baseline specification, no window of four or more consecutive years passes. Across the capital-price and input-set specifications considered, no window of five or more consecutive years passes. Allowing the common degree to vary freely therefore does little to reconcile even relatively short sequences of annual observations with the maintained model.

Selecting observations produces a different picture from shortening the calendar horizon. A maximum-cardinality rationalizable subset contains around five years for the median industry, but those years span a median of 43 calendar years; their longest consecutive run has median length one. No industry has a maximum-cardinality rationalizable subset composed entirely of consecutive years. Thus, the existence of a small consistent subset does not establish the existence of a short period over which the model holds. Moreover, maximum-cardinality subsets are typically not unique, and the implied returns-to-scale classifications can depend on which subset is selected. Recovering a degree from one such subset requires care in interpreting it as a property of the industry.

The efficiency index shows that the near-universal rejection over longer windows conceals substantial variation in the magnitude of departure. The median cost-efficiency slack is 2.4% over three years, 13.5% over a decade, and 27.7% over the full panel. Exact consistency thus becomes rare well before the relaxation required to restore it becomes large. Reporting both the exact test and the efficiency index separates these two features of the evidence.

The empirical analysis also documents variation over time and across industries. The share of adjacent year pairs passing the test falls by about 3.8 percentage points per decade, from roughly 80% near the beginning of the sample to below 45% near its end. Consistency is weaker in industries with faster capital deepening and more volatile output, and pairwise failure is associated with movements in output not matched by movements in measured inputs. These are descriptive patterns. They are compatible with changing utilization, adjustment frictions, technology change, and measurement problems, but the test does not identify which mechanism generates them. Nor does the absence of a difference between recession- and expansion-dated windows rule out other forms of cyclical variation.

1.1 Related literature

The paper builds on revealed-preference analysis, beginning with Afriat (1967), and on nonparametric production tests (Hanoch and Rothschild, 1972; Diewert and Parkan, 1983; Varian, 1984); see also Forges and Minelli (2009) and Nishimura et al. (2017) for broader treatments of rationalizability. Within this tradition, our object is the set of common homogeneity degrees compatible with observed-price cost minimization. The sharp bounds express the information in the finite dataset as an identified set rather than a selected parameter value (Manski, 2003).

Silva and Stefanou (1996) develop nonparametric tests of homothetic production with freely transformed output levels. We study homogeneity in observed output units, so output ratios identify the admissible common degrees. This yields a sharp, closed-form identified set and explicit rejection certificates, which we use to assess model fit and the sensitivity of returns-to-scale classifications to sample selection. The distinction between the two production restrictions is formalized in Remark 2.

Liu and Wong (2000) and Cavaignac and Briec (2007) examine the related questions of homotheticity and inverse homotheticity. More recently, Boussemart et al. (2024) develop a nonparametric Λ-returns-to-scale framework with observation-specific scale properties. Their allowance for heterogeneity addresses a complementary question to our requirement that one homogeneous technology rationalize repeated observations for a given industry.

Our measures of departure also draw on established approaches to goodness of fit. The cost-efficiency index adapts the logic of Afriat (1972, 1973) and Varian (1985, 1990) to homogeneous cost comparisons. The maximal-consistent-subset measure follows Houtman and Maks (1985); its general computational difficulty is studied by Smeulders et al. (2014, 2021). Other measures include those of Echenique et al. (2011), Apesteguia and Ballester (2015), and Dean and Martin (2016), while Halevy et al. (2018) connect recoverability and fit in consumer demand. Here, the scalar structure of the common-degree problem yields a tractable characterization of efficiency that can be applied repeatedly across industries and observation windows.

The exercise complements parametric production and cost models (Cobb and Douglas, 1928; Christensen et al., 1973), production-function estimation with proxy or control-function methods (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg et al., 2015; Gandhi et al., 2020; Blundell and Bond, 2000), and frontier approaches (Charnes et al., 1978; Banker et al., 1984; Deprins et al., 1984). These methods differ in their behavioral and technological assumptions; our findings should not be read as a rejection of all of them. In particular, approaches allowing unobserved productivity heterogeneity (Cherchye et al., 2018) need not impose the common-technology restriction tested here. Unlike statistical frontier tests of returns to scale (Simar and Wilson, 2002), our exercise is deterministic and conditional on the observed dataset. Developing sampling theory for these diagnostics remains outside the present analysis.

Finally, the empirical patterns connect to work on utilization and measured returns to scale (Hall, 1988; Basu, 1996; Burnside et al., 1995; Basu et al., 2006), and to the broader productivity literature (Syverson, 2011; Foster et al., 2008). The contribution is to document where a stable homogeneous representation fails without choosing a parametric production function, while preserving the distinction between detecting that failure and explaining its source.

The remainder of the paper proceeds as follows. Section 2 develops the fixed-degree test, identification bounds, and rejection certificates. Section 3 studies cost-efficiency and deletion measures. Sections 4.1 and 4 describe the data and present the empirical evidence. Section 5 discusses interpretation and limitations, and Section 6 concludes. The appendices contain the proofs, the multi-output extension, data construction details, and additional empirical results.

2 Framework

2.1 Setup

At each period t we observe a producer choosing an input vector x_t, facing input prices w_t, and producing output y_t. With L inputs,

$$w_t \in \mathbb{R}^L_{++}, \qquad x_t \in \mathbb{R}^L_+ \setminus \{\mathbf{0}\}, \qquad y_t \in \mathbb{R}_{++}.$$

The dataset is $S = \{(y_t, x_t, w_t)\}_{t\in\mathbb{T}}$ with $\mathbb{T} = \{1, \ldots, T\}$ finite. It is convenient to work with normalized prices

$$v_t := \frac{w_t}{w_t \cdot x_t}, \qquad \text{so that } v_t \cdot x_t = 1 \text{ for all } t.$$

This is a scale-free transformation that imposes no restriction on the data; $w_t \cdot x_t > 0$ because $x_t \neq \mathbf{0}$ and $w_t \gg 0$.

Definition 1 (Cost rationalization)**.** A production function $f : \mathbb{R}^L_+ \to \mathbb{R}_+$ *cost-rationalizes* S if, for each $t \in \mathbb{T}$,

$$f(x_t) = y_t \qquad \text{and} \qquad f(x) \geq f(x_t) \;\;\Rightarrow\;\; w_t \cdot x \geq w_t \cdot x_t.$$

That is, x_t minimizes cost among input bundles capable of producing at least y_t. A production function *exhibits returns to scale* k if it is homogeneous of degree k: $f(\lambda x) = \lambda^k f(x)$ for all $\lambda > 0$. Degree $k = 1$ is constant returns, $k > 1$ increasing, $k < 1$ decreasing.

The benchmark is Varian's (1984) characterization: S is cost-rationalizable by a homogeneous of degree one technology if and only if $v_s \cdot x_t \geq y_t/y_s$ for all $s, t \in \mathbb{T}$. This gives the degree-one benchmark. We use an output transformation to express the corresponding restrictions at any positive common degree and then solve them for the set of admissible degrees.

2.2 Testing a given degree of homogeneity

Theorem 1 (Test at a given degree of returns to scale)**.** *Fix $k > 0$ and consider the dataset $S = \{(y_t, x_t, w_t)\}_{t\in\mathbb{T}}$. The following are equivalent:*

1. *There exists a monotone, degree-k homogeneous production function that cost-rationalizes S.*

2. *For all* $s, t \in \mathbb{T}$,

$$v_s \cdot x_t \;\geq\; \left(\frac{y_t}{y_s}\right)^{1/k}. \tag{1}$$

where $v_s = w_s / (w_s \cdot x_s)$.

3. *There exists a continuous, monotone,* quasi-concave*, degree-*k *homogeneous production function that cost-rationalizes* S.

Moreover, whenever these hold, the min-envelope

$$\tilde{h}(x) \;:=\; \min_{t \in \mathbb{T}} y_t \big(v_t \cdot x\big)^k, \qquad x \in \mathbb{R}^L_+, \tag{2}$$

is continuous, monotone, quasi-concave, and homogeneous of degree k*, and cost-rationalizes* S*. It is concave if and only if* $k \in (0, 1]$.

Proof. See Appendix A. □

Statement 3 records quasi-concavity explicitly, and this is not a cosmetic strengthening. Concavity of the rationalizing technology is *impossible* under increasing returns, as Remark 4 below records, so the natural regularity property to ask for at $k > 1$ is quasi-concavity—convex input requirement sets, equivalently convex isoquants—which is a standard regularity condition in applied production analysis and which $\tilde{h}$ possesses at every $k > 0$. The theorem therefore delivers a well-behaved rationalizer across the whole range of returns to scale, not only the decreasing-returns range.

Remark 1 (Relation to Varian, 1984)**.** We state plainly what is and is not new. A function f is homogeneous of degree k if and only if $g := f^{1/k}$ is homogeneous of degree one, and $f(x) \geq y \Leftrightarrow g(x) \geq y^{1/k}$, so the two cost-minimization problems have identical feasible sets. Applying Varian's constant-returns test to $\{(y_t^{1/k}, x_t, w_t)\}$ therefore yields (1) directly, and $\tilde{h}$ is the degree-k analogue of Varian's linear envelope. Theorem 1 is thus best read as a change of variable applied to a known result, together with the quasi-concavity statement, and we make no stronger claim for it. Its role here is instrumental: because k enters (1) only through the exponent, the unknown-k problem inherits a one-dimensional structure, and that structure—not the fixed-k test itself—is what delivers the identification, falsification, and efficiency results that follow, and what makes the empirical exercise of Section 4 feasible at panel scale.

Remark 2 (Homotheticity and homogeneity of observed output)**.** Theorem 1 of Silva and Stefanou (1996, p. 543) uses positive numbers a^t, increasing in observed output, and a degree

$r > 0$. With indices relabeled, its inequalities are

$$v_s \cdot x_t \geq (a^t/a^s)^{1/r} \quad \text{for all } s, t.$$

Here a^t represents the value of a homogeneous index, not necessarily the observed output y_t. The substitution $h_t = (a^t)^{1/r}$ gives $v_s \cdot x_t \geq h_t/h_s$, with the same output ordering. More explicitly, if (a^t, r) satisfies this system, then, for any $r' > 0$, $a'^t = (a^t)^{r'/r}$ preserves all inequalities and the ordering of the latent levels. This invariance concerns the ordinal index in that system; it does not identify a physical returns-to-scale parameter.

Our maintained hypothesis fixes $a^t = y_t$ and $r = k$, or equivalently $h_t = y_t^{1/k}$. This links the normalization of the homogeneous index to observed output ratios. If $H = g \circ f$ is homogeneous of degree r and g is unrestricted, the homogeneity of H does not imply that the observed-output function f is homogeneous. In the differentiable case, its radial elasticity is

$$\left.\frac{d \log f(\lambda x)}{d \log \lambda}\right|_{\lambda=1} = \frac{r\, g(f(x))}{f(x) g'(f(x))},$$

where $f(x) > 0$ and $g'(f(x)) > 0$; it need not be constant. No differentiability is required for either of the finite-data tests being compared.

For a strict distinction, take one input, $w_t = 1$, inputs $(x_1, x_2, x_3) = (1, 2, 4)$, and outputs $(y_1, y_2, y_3) = (1, 2, 8)$. The increasing function $f(x) = x$ for $0 \leq x \leq 2$ and $f(x) = 3x - 4$ for $x \geq 2$ cost-rationalizes these observations and is homothetic: its inverse transforms it into the degree-one index $H(x) = x$. The latent levels $a^t = x_t$ therefore satisfy the homotheticity inequalities at $r = 1$. However, the first two observations require $k = 1$ in our model, whereas the last two require $k = 2$; hence $\mathcal{K}(S) = \varnothing$. Rejection of our common-degree model does not establish rejection of general homotheticity.

Silva and Stefanou also propose searching over r in program (T.2) on p. 544. As displayed there, the free levels a^i enter through $(a^i)^{1/r}$, including the normalization $(a^1)^{1/r} = w^1 \cdot x^1$. Replacing these quantities by h_i removes r from that program as well. Consequently, without additional restrictions fixing the output transformation, its feasibility cannot be interpreted as bounds on the degree of the observed-output production function studied here. Our closed-form bounds concern that latter parameter under the explicit normalization $a^t = y_t$. This is a restriction on the production model, not a stronger test of unrestricted homotheticity.

Remark 3 (Equivalent dual cost restrictions at a common degree)**.** Fix $k > 0$ and define the output-normalized input bundles and costs by

$$z_t := \frac{x_t}{y_t^{1/k}}, \qquad c_t := w_t \cdot z_t = \frac{w_t \cdot x_t}{y_t^{1/k}}.$$

Homogeneity of degree k implies the cost scaling relation $C(w, y) = y^{1/k} c(w)$, where $c(w) := C(w, 1)$. If the observations minimize costs, z_t is a unit-output cost-minimizing bundle at w_t. It consequently supports the concave unit-output cost function:

$$c(w) \leq c_t + z_t \cdot (w - w_t) = w \cdot z_t, \qquad c(w_t) = c_t.$$

Evaluating at each observed price gives

$$c_s \leq w_s \cdot z_t \quad \text{for all } s, t. \tag{3}$$

Substituting the definitions and multiplying by $y_t^{1/k}$ yields

$$w_s \cdot x_t \geq (w_s \cdot x_s)(y_t/y_s)^{1/k},$$

which is exactly (1). Conversely, if these inequalities hold, the function

$$\widehat{c}(w) := \min_{t \in \mathbb{T}} w \cdot z_t$$

is continuous, nondecreasing, concave, and homogeneous of degree one in prices; it satisfies $\widehat{c}(w_s) = c_s$ at every observation and has z_s as a supergradient there. Theorem 1 supplies a production rationalizer for the same observations. Hence the finite supporting restrictions of this dual common-degree formulation and the primal pairwise test have exactly the same empirical content. No differentiability or uniqueness of conditional input demands is needed. The closed-form bounds in Proposition 2 solve this same system for the unknown common degree; their primal presentation does not by itself create additional identifying information.

Remark 4 (Concavity and increasing returns)**.** Concavity is incompatible with homogeneity of degree $k > 1$ on $\mathbb{R}^L_+$ under monotonicity and $f(\mathbf{0}) = 0$. Applying concavity between x and $\mathbf{0}$ gives $f(\theta x) \geq \theta f(x)$ for $\theta \in (0, 1)$, while homogeneity gives $f(\theta x) = \theta^k f(x)$ with $\theta^k < \theta$; together these force $f(x) \leq 0$, and monotonicity with $f \geq f(\mathbf{0}) = 0$ then forces $f \equiv 0$. Increasing returns therefore rule out a concave rationalizer as a matter of logic, not of technique. Quasi-concavity is the appropriate substitute, and Theorem 1 supplies it.

Proposition 1 (The canonical rationalizer is pointwise maximal)**.** *Fix $k > 0$ and suppose S is rationalizable by a monotone, degree-k homogeneous production function. Then for every* *monotone, degree-k homogeneous f that cost-rationalizes S,*

$$f(x) \leq \tilde{h}(x) \qquad \text{for all } x \in \mathbb{R}^L_+.$$

Proof. See Appendix A. □

The min-envelope is therefore not merely one convenient rationalizer: it is the largest monotone degree-k technology consistent with the data, and so delivers a sharp upper bound on counterfactual output at any input bundle.

2.3 Identifying returns to scale

When k is not fixed a priori, (1) identifies a set of returns-to-scale parameters. Write, for each ordered pair (s,t),

$$a_{st} := v_s \cdot x_t, \qquad q_{st} := y_t / y_s,$$

so that the pairwise restriction reads $a_{st} \geq q_{st}^{1/k}$, and let

$$\mathcal{K}(S) := \left\{ k > 0 : \ a_{st} \geq q_{st}^{1/k} \ \text{ for all } s,t \in \mathbb{T} \right\}.$$

Proposition 2 (Sharp identified set of homogeneity degrees)**.** *If some ordered pair satisfies either*

$$q_{st} > 1 \text{ and } a_{st} \leq 1, \qquad \text{or} \qquad q_{st} = 1 \text{ and } a_{st} < 1,$$

then $\mathcal{K}(S) = \varnothing$. Otherwise, with the conventions $\max \varnothing := 0$ and $\min \varnothing := +\infty$, define

$$\underline{k}(S) := \max_{(s,t):\, q_{st}>1,\ a_{st}>1} \frac{\ln q_{st}}{\ln a_{st}}, \qquad \overline{k}(S) := \min_{(s,t):\, q_{st}<1,\ a_{st}<1} \frac{\ln q_{st}}{\ln a_{st}}.$$

Then $\mathcal{K}(S) = (0,\infty) \cap [\underline{k}(S), \overline{k}(S)]$. In particular $\mathcal{K}(S) \neq \varnothing$ if and only if $\underline{k}(S) \leq \overline{k}(S)$, and returns to scale are point identified if and only if $\underline{k}(S) = \overline{k}(S)$.

Proof. See Appendix A. □

Two features of the statement are worth emphasizing for the empirical work. First, $\mathcal{K}(S)$ is available in closed form from $O(T^2)$ elementary operations, so testing is trivial even for the tens of millions of subsets examined in Section 4. Second, the restrictions are linear in $\theta := 1/k$; the identified set is therefore an interval in θ and, by monotonic transformation, an interval in $\log k$. When a scalar summary is useful, we report the geometric midpoint of a bounded $\mathcal{K}(S)$ as a log-symmetric descriptive summary; it is not implied by linearity in $1/k$, and all identification statements use the full interval.

When $\mathcal{K}(S) \neq \varnothing$ we summarize its position relative to constant returns by

$$\rho(S) := \inf_{k \in \mathcal{K}(S)} |k-1|, \tag{4}$$

the minimal departure from constant returns among exactly rationalizing degrees, with $\rho(S) = 0$ if and only if $1 \in \mathcal{K}(S)$. We stress that $\rho(S)$ measures distance from the constant-returns *benchmark*, not instability of the technology: if $\mathcal{K}(S) = \{1.8\}$, a single perfectly stable degree-1.8 technology rationalizes every observation exactly, yet $\rho(S) = 0.8$. When $\mathcal{K}(S) = \varnothing$, $\rho(S)$ is not defined, and the measures of Section 3 take over.

Because $\mathcal{K}(S)$ is an intersection of one-dimensional intervals, its emptiness has an unusually simple structure.

Theorem 2 (Sparse rejection certificate)**.** *Let $\mathcal{K}_{st} \subseteq (0, \infty)$ denote the set of k satisfying $a_{st} \geq q_{st}^{1/k}$, so $\mathcal{K}(S) = \bigcap_{s,t} \mathcal{K}_{st}$. Then $\mathcal{K}(S) = \varnothing$ if and only if at least one of the following holds:*

1. *there is a single ordered pair (s,t) with $\mathcal{K}_{st} = \varnothing$;*
2. *there are two ordered pairs (s,t) and (p,q) with $\mathcal{K}_{st} \cap \mathcal{K}_{pq} = \varnothing$.*

Every rejection of stable homogeneous cost rationalization is therefore witnessed by at most two ordered pairs of observations.

Proof. See Appendix A. □

Theorem 2 makes rejection localizable. A researcher who finds $\mathcal{K}(S) = \varnothing$ can always exhibit, at cost linear in the number of pairs, either a single comparison that no degree of homogeneity reconciles with cost minimization, or two comparisons whose implied bounds on k are mutually inconsistent. The sparsity itself is the one-dimensional Helly property; what is specific here is the explicit description of each $\mathcal{K}_{st}$ in terms of the observables q_{st} and a_{st}, which turns an abstract intersection property into a constructive and economically readable certificate.

The two mechanisms are economically distinct, and we track both empirically. Let

$$\mathcal{I}(S) := \big\{(s,t) : q_{st} > 1,\ a_{st} \leq 1\big\} \cup \big\{(s,t) : q_{st} = 1,\ a_{st} < 1\big\}$$

denote the *directly impossible* pairs, those excluded for every $k > 0$ on their own. A directly impossible pair says that, at another year's prices, a bundle producing strictly more output is no more expensive, or a bundle producing the same output is strictly cheaper—an outright violation of cost minimization at any scale. When $\mathcal{I}(S) = \varnothing$ but $\mathcal{K}(S) = \varnothing$, rejection instead comes from $\underline{k}(S) > \overline{k}(S)$: two individually unremarkable comparisons whose implied ranges for k do not overlap, which is a violation of *stability* rather than of cost minimization as such. Section 4 reports the incidence of each.

Two further properties are used repeatedly below. We state them as observations rather than as formal propositions, since both are immediate.

Observation 1 (Monotonicity)**.** If $S \subseteq S'$ then $\mathcal{K}(S') \subseteq \mathcal{K}(S)$: adding observations can only shrink the set of admissible degrees. Consequently $\rho(S') \geq \rho(S)$ whenever both are defined. This is immediate, because $\mathcal{K}(S')$ is defined by a superset of the inequalities defining $\mathcal{K}(S)$.

Observation 2 (Feasibility is downward closed)**.** If $A \subseteq \mathbb{T}$ satisfies $\mathcal{K}(S_A) \neq \varnothing$, then $\mathcal{K}(S_B) \neq \varnothing$ for every $B \subseteq A$; equivalently, if $\mathcal{K}(S_A) = \varnothing$ then $\mathcal{K}(S_{A'}) = \varnothing$ for every $A' \supseteq A$. This is the contrapositive of Observation 1.

Observation 2 does the computational work in Section 4. Because infeasibility propagates to supersets, the feasible subsets of $\mathbb{T}$ form a downward-closed family, which is exactly the structure branch-and-bound search exploits: any partial subset found infeasible is pruned immediately, and feasibility of a candidate A is checked in $O(|A|^2)$ time by Proposition 2. In the consecutive-window exercise it is sharper still: for a fixed start date, feasibility is monotone in window length, so a single failing window terminates the search over all longer windows with the same start.

Finally, the two-observation case, which is the elementary building block of everything above, is worth recording explicitly.

Observation 3 (Two observations)**.** For $S = \{(y_1, x_1, w_1), (y_2, x_2, w_2)\}$ with $y_1 \neq y_2$, only the two cross-observation inequalities $v_1 \cdot x_2 \geq (y_2/y_1)^{1/k}$ and $v_2 \cdot x_1 \geq (y_1/y_2)^{1/k}$ bind. By Proposition 2 one of them contributes at most a lower bound and the other at most an upper bound, so $\mathcal{K}(S)$ is either empty or an interval determined by at most one bound of each type, point identified exactly when the two coincide. Two observations can already deliver a nondegenerate identified interval; what requires at least three observations is the possibility of competing bounds of the same type coming from different comparisons.

Example 1 (Two observations: decreasing and increasing returns)**.** Let $w_1 = w_2 = (5, 5)$, $x_1 = (10, 10)$, $x_2 = (20, 20)$, so $v_1 = (0.05, 0.05)$, $v_2 = (0.025, 0.025)$, and $v_1 \cdot x_2 = 2$, $v_2 \cdot x_1 = 0.5$. Both restrictions bind at the same $k^\star$ solving $2 = (y_2/y_1)^{1/k}$, that is $k^\star = \ln(y_2/y_1)/\ln 2$. With $y_1 = 100$ and $y_2 = 150$, $k^\star = \ln(3/2)/\ln 2 \approx 0.585$: decreasing returns, $\rho(S) \approx 0.415$. With $y_2 = 300$, $k^\star = \ln 3/\ln 2 \approx 1.585$: increasing returns, $\rho(S) \approx 0.585$. Note that k is point identified here, an artifact of the two observations lying on a ray; generically the data deliver an interval.

Example 2 (Four periods, increasing returns)**.** Let $w_t = (5,5)$ for all t, $x_t = (10 \cdot 2^{t-1}, 10 \cdot 2^{t-1})$ and $y_t = 100{\cdot}3^{\,t-1}$ for $t = 1, \dots, 4$, so doubling inputs triples output. Then $v_t{\cdot}x_s = 2^{\,s-t}$ and $y_s/y_t = 3^{\,s-t}$, so every pairwise restriction reads $2^{\,s-t} \geq 3^{(s-t)/k}$; taking logarithms and dividing by $s{-}t$ shows each binds with equality at the same $k^\star = \ln 3/\ln 2 \approx 1.585$, regardless of the sign of $s-t$. Hence $\mathcal{K}(S) = \{1.585\}$ and $\rho(S) \approx 0.585$: exact point identification survives with more than two observations when the data lie exactly on a homogeneous technology. Our replication code reproduces this example to twelve digits as a regression test.

3 Measuring departures from stable homogeneous rationalization

An exact test returns a verdict, not a magnitude. When $\mathcal{K}(S) = \varnothing$ for every unit in a panel, the verdict alone is uninformative, and some graded measure is required. There are two economically distinct ways a dataset can fail, and we retain exactly one measure of each, resisting the temptation to report more.

- **Intensive margin.** By how much must each cost-minimization inequality be relaxed before a single admissible degree rationalizes *all* observations? This is measured by the cost-efficiency index $e^*(S;\mathbb{K})$ of Section 3.1.
- **Extensive margin.** How many observations must be discarded before the remainder is exactly rationalizable? This is measured by the deletion distance $d_N(S)$ of Section 3.2.

3.1 The cost-efficiency index

The pairwise restriction $v_s{\cdot}x_t \geq (y_t/y_s)^{1/k}$ compares two positive quantities multiplicatively, so the natural relaxation is multiplicative as well. Fix a compact, economically admissible parameter set $\mathbb{K} = [k_{\min}, k_{\max}] \subset (0,\infty)$, carried explicitly in the notation throughout.

Definition 2 (e-approximate homogeneous cost rationalization)**.** For $e \in (0,1]$, the dataset S is *e-rationalizable at degree k* if

$$w_s \cdot x_t \;\geq\; e\,(w_s \cdot x_s)\left(\frac{y_t}{y_s}\right)^{1/k} \qquad \text{for all } s,t \in \mathbb{T}. \tag{5}$$

Condition (5) says that the observed bundle at s costs no more than a factor $1/e$ above the cost of the cheapest scaled bundle from t capable of producing y_s: the producer is within a fraction e of minimizing cost at every comparison. Exact rationalization is $e = 1$.

Definition 3 (Homogeneous cost-efficiency index)**.**

$$e^*(S;\mathbb{K}) := \max_{k\in\mathbb{K}} \min_{s,t\in\mathbb{T}} \frac{v_s \cdot x_t}{(y_t/y_s)^{1/k}}.$$

Thus $e^*(S;\mathbb{K})$ is the largest e for which S is e-rationalizable at some common admissible degree, and $1 - e^*(S;\mathbb{K})$ is the proportional cost-efficiency allowance required to reconcile the data with the model. It is a critical cost-efficiency index(CCEI)-style slack parameter, not a literal estimate of productive waste. It is the exact production analogue of Afriat's critical cost-efficiency index (Afriat, 1972, 1973; Varian, 1990).

Remark 5 (Relation to proportional and additive slack)**.** Writing

$$D_{\mathrm{rel}}(S;\mathbb{K}) := \min_{k\in\mathbb{K}} \max_{s,t} \big[(y_t/y_s)^{1/k}/(v_s \cdot x_t) - 1\big]_+ \tag{6}$$

for the proportional violation, one has

$$e^*(S;\mathbb{K}) = 1/\big(1 + D_{\mathrm{rel}}(S;\mathbb{K})\big), \tag{7}$$

so the two carry identical information. The normalized additive slack

$$\big[(y_t/y_s)^{1/k} - v_s \cdot x_t\big]_+$$

is also invariant to changes in input units, output units, and period-specific price rescalings, because both terms are already normalized and unchanged by these transformations. Unit invariance therefore does not distinguish the additive and multiplicative approaches. We report e^* because it has a proportional interpretation, is bounded in $(0, 1]$, and admits the convenient logarithmic optimization representation in Proposition 3. The normalized additive slack measures a difference rather than a ratio and is not generally bounded by one.

Proposition 3 (Properties and computation of e^*)**.** *Let $\mathbb{K} = [k_{\min}, k_{\max}] \subset (0,\infty)$ be compact. Then:*

1. $e^*(S;\mathbb{K}) \in (0, 1]$*, and the maximum is attained.*
2. $e^*(S;\mathbb{K}) = 1$ *if and only if* $\mathcal{K}(S) \cap \mathbb{K} \neq \varnothing$*. In particular* $e^*(S;\mathbb{K}) < 1$ *whenever the exact test rejects for every admissible degree.*

3. *If $S \subseteq S'$ then $e^*(S';\mathbb{K}) \le e^*(S;\mathbb{K})$: the index is monotone under data expansion.*

4. *$e^*(S;\mathbb{K})$ is invariant to rescaling any input's units, to rescaling output units, and to rescaling each period's prices.*

5. *With $\theta := 1/k$,*

$$\log e^*(S;\mathbb{K}) = \max_{\theta \in [1/k_{\max},\, 1/k_{\min}]} \min_{s,t \in \mathbb{T}} \big(\log a_{st} - \theta \log q_{st}\big),$$

which is the maximum of a concave, piecewise-linear function of a single variable, equivalently the linear program $\max\{z :\ z \le \log a_{st} - \theta \log q_{st}\ \forall s,t;\ \theta \in [1/k_{\max}, 1/k_{\min}]\}$.

Proof. See Appendix A. □

Part 5 is what makes the index practical. Because the objective is concave in θ, the optimum is global and can be computed to arbitrary numerical precision by one-dimensional concave optimization; equivalently, it is characterized exactly by the two-variable linear program. There is no grid and no risk of a local optimum. This matters at panel scale: the index is computed here for 361 industries over samples with up to 3,422 ordered pairs each, and again for every one of the 180,000-plus windows in Section 4. Part 3 is the property that a sample-average measure of violations lacks: averaging pairwise slack over $T(T-1)$ pairs makes the denominator grow with the sample, so adding well-behaved observations can lower the measure even though the dataset has become strictly harder to rationalize. Part 2 guarantees that e^* never certifies rationalizability that does not exist, a failure mode that any measure optimizing k separately for each pair does exhibit, since every pair can be individually rationalizable by *some* degree while no single degree rationalizes them all.

Remark 6 (The role of $\mathbb{K}$)**.** e^* depends on $\mathbb{K}$ and the notation says so. The dependence is monotone and transparent: $\mathbb{K} \subseteq \mathbb{K}'$ implies $e^*(S;\mathbb{K}) \le e^*(S;\mathbb{K}')$, and $e^*(S;\mathbb{K})$ is non-decreasing as $\mathbb{K}$ widens, with a well-defined limit as $\mathbb{K} \uparrow (0,\infty)$. In the application we set $\mathbb{K} = [0.01, 100]$, a deliberately wide numerical domain. Because some exact identified sets can be one-sided or unbounded, this interval should not be described as containing every $\mathcal{K}(S)$. The reported e^* values are therefore formally relative to $\mathbb{K}$; endpoint binding of the maximizing k should be reported as a numerical diagnostic. A researcher who wishes to impose economically motivated bounds—for instance $\mathbb{K} = [0.5, 2]$—may do so, and the index then answers the correspondingly narrower question.

3.2 The deletion distance

The complementary question is extensive: how much of the sample must be abandoned rather than how far each inequality must be bent.

Definition 4 (Minimum deletion distance)**.** Let $m^*(S) := \max\{|A| : A \subseteq \mathbb{T},\ \mathcal{K}(S_A) \neq \varnothing\}$ be the size of the largest rationalizable subset, where S_A denotes S restricted to A. The *minimum deletion distance* is

$$d_N(S) := \frac{|\mathbb{T}| - m^*(S)}{|\mathbb{T}|} \in [0, 1].$$

$d_N(S)$ is the smallest fraction of observations that must be discarded before some homogeneous technology rationalizes the remainder; $d_N(S) = 0$ if and only if S is rationalizable. It is the production analogue of the Houtman and Maks (1985) index, and inherits that index's computational difficulty: the underlying maximum-feasible-subset problem is non-deterministic polynomial time-hard in general (Smeulders et al., 2014). It is tractable here because Observation 2 makes branch-and-bound pruning effective and because feasibility of a candidate subset is checked in closed form; Section 4 solves it exactly for every industry. We call any subset $A^* \subseteq \mathbb{T}$ with $\mathcal{K}(S_{A^*}) \neq \varnothing$ and $|A^*| = m^*(S)$ a maximizer; that is, a maximizer is a largest rationalizable subset of observations, and $m^*(S)$ is its common size.

Two cautions apply to m^* and d_N, and we take both in the empirical work. First, the maximizing subset need not be unique, and distinct maximizers may imply different identified sets. Any returns-to-scale classification built on one arbitrarily selected maximizer is therefore not selection-invariant, and we report instead the envelope of $\mathcal{K}$ over *all* maximizers, together with the classifications that are invariant. Second, m^* places no restriction on *which* observations are retained. A large m^* achieved by non-adjacent years scattered across decades says something quite different from the same m^* achieved by a consecutive block, and only the latter supports a claim of local-in-time stability. Section 4 accordingly reports the consecutive-window analogue of m^* separately, and the two turn out to differ sharply.

4 Empirical Implementation: U.S. Manufacturing Industries

This section applies the revealed-preference tests to U.S. manufacturing industries using the NBER-CES Manufacturing Industry Database (Becker et al., 2021). We examine whether observed input choices are consistent with cost minimization under a homogeneous production technology and identify the degrees of homogeneity that satisfy the tests. The analysis does not require a parametric production function.

We first describe the data, then report exact tests for the full panel and for consecutive windows. We next examine the largest rationalizable subsets and use the cost-efficiency index to measure departures from the model. Finally, we study changes over time, differences across industries, and sensitivity to the treatment of capital.

4.1 Data

We apply the framework to the NBER-CES Manufacturing Industry Database (Becker et al., 2021), 2012 NAICS vintage, which reports annual U.S. manufacturing data on output, employment, payroll, production-worker hours, material and energy costs, investment, capital stocks, and industry-specific deflators. The public database covers 1958–2018; we use 1958–2016 because the real capital stock is available through 2016. We drop three industries with problematic definitions in the construction file, leaving a balanced panel of 361 industries observed for 59 years, or 21,299 industry-years.

Output is real gross output, $y_{it} = \mathrm{VSHIP}_{it}/\mathrm{PISHIP}_{it}$. Gross output rather than value added is appropriate because the revealed-preference inequalities compare the cost of producing an observed output level using all measured inputs. The input vector follows the five-factor structure used in the NBER-CES productivity measures, $x_{it} = (K_{it}, L_{it}, N_{it}, M_{it}, E_{it})$, where K is the real capital stock, L production-worker hours, N non-production employment, E real energy use, and M real non-energy materials.

The test does not require prices in absolute units, only the normalized cross-cost terms, which are recoverable from expenditures: with e^j_{it} expenditure on input j and $C_{it} = \sum_j e^j_{it}$,

$$v_{is} \cdot x_{it} = \sum_j \frac{e^j_{is}}{C_{is}} \frac{x^j_{it}}{x^j_{is}}, \tag{8}$$

so the test uses only expenditure shares and input-quantity ratios. The treatment of capital affects all normalized prices through total expenditure. Since the NBER-CES database does not report a rental price of capital, we construct a Jorgensonian user cost (Jorgenson, 1963; Hall and Jorgenson, 1967),

$$w_{K,it} = P_{I,it}\big(r + \delta_i - \pi^K_i\big), \qquad e^K_{it} = w_{K,it} K_{it}, \tag{9}$$

with δ_i and π^K_i built from the same perpetual-inventory identity and investment deflator that NBER-CES itself uses, following standard multifactor-productivity practice (Jorgenson et al., 1987; OECD, 2009). The baseline sets $r = 8\%$; $r \in \{6\%, 10\%\}$ is reported as a sensitivity band, and Section 4.7 also reports a four-input specification that excludes capital from the priced input vector entirely. Full details, including the distributions of δ_i and π^K_i, are in Appendix B.

Unit of observation. The theory is written for a producer observed repeatedly; the data are industry-year. We therefore interpret a six-digit manufacturing industry as the empirical decision-making unit, treating $S_i = \{(y_{it}, x_{it}, w_{it})\}$ as repeated observations on an industry-level technology. This is appropriate if the industry data are read as the choices of a representative cost-minimizing producer, or as the aggregate outcome of establishments operating under a common industry technology.

The distinction matters for interpretation, and we return to it in Section 5. A rejection does not by itself show that individual plants fail to minimize costs. Industry aggregates combine heterogeneous establishments, changing product mixes, quality change, entry and exit, and time-varying productivity, and aggregation alone can break the exact restrictions even when every plant optimizes. What the test asks is whether the aggregate industry series behaves as if generated by a stable homogeneous technology and cost-minimizing input choices. The test imposes strong restrictions on applications that pool observations under a common production structure; many growth-accounting and production-function methods allow productivity shifters or other forms of time variation, so rejection here should not be read as rejection of those broader models.

4.2 Exact tests over long panels

Table 1 reports the exact-rationalization results. Over the full 1958–2016 panel, none of the 361 industries has a nonempty identified set. This is not driven by a single anomalous year: removing each year in turn, across all 21,299 industry-year deletions, never restores a nonempty $\mathcal{K}(S_i)$ over the remaining years. Even over the three-year 2014–2016 panel, only one industry passes, and its identified set excludes constant returns.

Which mechanism drives rejection? Theorem 2 distinguishes two sources of rejection: a pair that is inconsistent with every degree of homogeneity, and pairs whose bounds cannot be satisfied by a common degree. Both occur in every industry. In the median industry, 145 of the 3,422 ordered pairs (4.2%) are directly impossible. Removing all such pairwise inequalities still leaves all 361 industries rejected: the remaining pairs imply $\underline{k}(S_i) > \overline{k}(S_i)$. Thus, rejection also reflects conflicting restrictions across years, rather than only individually impossible comparisons.

Table 1: Exact revealed-preference tests

Sample or exercise	Industries	Nonempty $\mathcal{K}(S_i)$	Empty $\mathcal{K}(S_i)$
1958–2016 full panel	361	0	361
Leave-one-year-out, 1958–2016	361	0	361
2007–2016 decade	361	0	361
2014–2016 short panel	361	1	360

Notes: All rows use the baseline specification, five inputs with capital priced at a user cost with $r = 8\%$ (Appendix B). The leave-one-year-out row removes each of the 59 years one at a time for every industry (21,299 industry-year removals in total) and reports how many industries have at least one single-year removal that restores a nonempty $\mathcal{K}(S_i)$. Section 4.7 shows the first two rows are unchanged for $r \in \{6\%, 8\%, 10\%\}$ and for the four-input specification.

4.3 How local in time is rationalizability?

We next examine whether the model fits over shorter periods. This is more relevant for many empirical applications than requiring a common technology over six decades. Consecutive windows allow us to assess how long the same homogeneous technology can rationalize an industry's data.

We therefore test every consecutive window. For each industry i, each window length L, and each start year τ, we compute $\mathcal{K}\big(S_i(\{\tau, \tau+1, \ldots, \tau+L-1\})\big)$ and record whether it is nonempty. Over $L \in \{2, 3, \ldots, 20\}$ this is more than 180,000 windows.

Table 2 and Figure 1 show a sharp decline in pass rates as the window length increases. Adjacent year pairs pass 61.9% of the time, and every industry has at least one passing pair. At three years the pass rate falls to 0.5%: only 106 of 20,577 three-year windows pass, spread over 83 industries, and only 20 industries have more than one such window. At four years the pass rate is exactly zero under the baseline specification, and it remains exactly zero at every longer length we test, up to twenty years.

The four-year result varies slightly across specifications (Table 7). The baseline has no passing four-year windows; the other specifications have at most two out of 20,216, or about one hundredth of one percent. No five-year window passes in any specification. Since adding observations cannot restore rationalizability, no longer consecutive window can pass either. Thus, no industry in this sample satisfies the exact restrictions over five or more consecutive years, or over four or more in the baseline.

Equivalently, define $L_i^{\max}$ as the longest run of consecutive years industry i can rationalize. Then $L_i^{\max} = 2$ for 278 industries (77.0%) and $L_i^{\max} = 3$ for the remaining 83 (23.0%); the mean is 2.23 and the maximum, over all 361 industries and six decades, is three. Among these longest subsets, 70.1% require increasing returns, 29.1% decreasing returns, and 0.8%

Table 2: Consecutive-window rationalizability, 361 industries, 1958–2016

Window	Windows		Pass	Industries with	Regime among passing windows		
length L	tested	passing	rate	≥ 1 pass	IRS	DRS	CRS
2	20,938	12,964	0.619	361	0.594	0.395	0.011
3	20,577	106	0.005	83	0.755	0.236	0.009
4	20,216	0	0.000	0	—	—	—
5	19,855	0	0.000	0	—	—	—
6	19,494	0	0.000	0	—	—	—
7	19,133	0	0.000	0	—	—	—
8	18,772	0	0.000	0	—	—	—
9	18,411	0	0.000	0	—	—	—
10	18,050	0	0.000	0	—	—	—
12	17,328	0	0.000	0	—	—	—
15	16,245	0	0.000	0	—	—	—
20	14,440	0	0.000	0	—	—	—

Notes: A window of length L starting in year τ passes if $\mathcal{K}(S_i) \neq \varnothing$ over the L consecutive years $\tau, \ldots, \tau + L - 1$. Every industry-start-year combination is tested. "Industries with ≥ 1 pass" counts industries having at least one passing window of that length anywhere in 1958–2016. Regime shares are computed among passing windows: IRS if $\mathcal{K}(S_i)$ lies strictly above one, DRS if strictly below, CRS if it contains one. Baseline specification, user cost of capital $r = 8\%$.

admit constant returns.

4.4 Largest rationalizable subsets and their non-uniqueness

We compute the largest rationalizable subsets to determine the deletion distance d_N. For each industry, we solve

$$m^*(S_i) = \max\{|T| : T \subseteq T_i,\ \mathcal{K}(S_i(T)) \neq \varnothing\} \tag{10}$$

exactly by branch and bound.

The median industry retains 5 years, the mean 5.22, the minimum 3 and the maximum 9. Correspondingly minimum deletion distance, d_N, has mean 0.912, median 0.915, and range $[0.847, 0.949]$: restoring exact rationalizability to a typical industry's panel requires discarding just over nine-tenths of its annual observations.

The largest rationalizable subsets contain around five years, but these years do not form a period of local stability. The selected subsets span a median of 43 calendar years, and their longest run of adjacent years has a median length of one. No industry has a maximum-size rationalizable subset made up of consecutive years. The number of retained observations

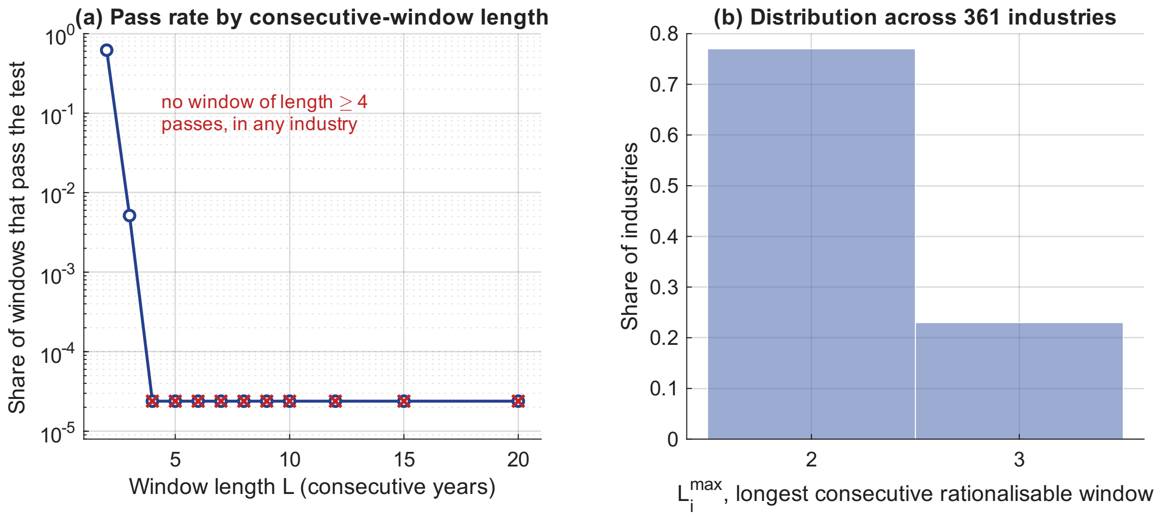

Figure 1: Consecutive-window rationalizability

Notes: Panel (a): share of windows with $\mathcal{K}(S_i) \neq \varnothing$, by window length, on a log scale; crosses mark lengths at which the pass rate is exactly zero. Panel (b): distribution across the 361 industries of $L_i^{\max}$, the longest consecutive window that industry i can rationalize. Baseline specification, user cost of capital $r = 8\%$.

and their proximity in time therefore describe different aspects of model fit.

One possible explanation involves short-run changes in capacity utilization, inventories, or input quality. Annual expenditure shares and the perpetual-inventory capital stock may not fully capture these changes (Basu, 1996; Burnside et al., 1995; Basu et al., 2006). The results in Section 4.6 are consistent with this interpretation. However, the selected subsets alone do not show that a randomly chosen distant pair is easier to rationalize than an adjacent pair.

Most industries have several rationalizable subsets of maximum size. Enumerating every maximum-cardinality rationalizable subset for every industry gives a median of 5 maximizers per industry, a mean of 13, an interquartile range of $[2, 15]$, and a maximum of 311. Only 62 industries (17.2%) have a unique maximizer.

This matters because different maximizers imply different identified sets. Table 3 contrasts the classification obtained from one arbitrarily selected maximizer with the selection-invariant classification obtained by requiring agreement across all of them. Using one selected maximizer yields 82.0% IRS, 17.5% DRS, and 0.6% CRS. Requiring agreement across all maximizers yields 66.5% IRS and 9.7% DRS, with 23.8% of industries classified as ambiguous because their maximum-size subsets imply different returns-to-scale regimes. No industry is CRS under every maximizer, while 8 admit constant returns under some maximizer.

The same point applies to the distribution of identified degrees. Figure 2 plots, for each

Table 3: Returns-to-scale classification from maximum feasible subsets: one maximizer versus all maximizers

	One selected maximizer		All maximizers agree	
Classification	Industries	Share	Industries	Share
IRS required	296	0.820	240	0.665
DRS required	63	0.175	35	0.097
CRS admissible	2	0.006	0	0.000
Ambiguous	—	—	86	0.238

Notes: Left panel classifies each industry using one maximum-cardinality rationalizable subset returned by the branch-and-bound solver. Right panel enumerates *every* maximum-cardinality rationalizable subset for every industry and classifies an industry only if all of them agree; otherwise the industry is "ambiguous". Enumeration is exact and complete for all 361 industries. Baseline specification, $r = 8\%$.

industry, the envelope of $\mathcal{K}(S_i(T))$ over all maximizers T against the interval delivered by a single selected maximizer. The median log-width of $\mathcal{K}$ from one maximizer is 0.0035; the median log-width of the envelope is 0.133, a factor of roughly 35 larger. Using only the midpoints from selected maximizers can therefore overstate precision. Across maximizers, identified degrees cluster modestly above one, and constant returns is admissible for very few industries under any selection.

The deletion distance d_N measures how much of the panel must be discarded, regardless of which maximizer is selected. Returns-to-scale classifications require more care: they should be reported across all maximizers, with disagreement made explicit. The consecutive-window results in Section 4.3 and the cost-efficiency index in Section 4.5 provide complementary measures of fit.

4.5 Measuring departures from the model

Every industry is rejected over the full panel, so exact tests cannot rank industries or horizons. The cost-efficiency index $e^*(S_i; \mathbb{K})$ of Definition 3 provides a quantitative measure of these departures: $1 - e^*$ is the proportional cost-efficiency slack that must be allowed before a single admissible degree of homogeneity rationalizes every comparison; it is not a literal estimate of productive waste. We set $\mathbb{K} = [0.01, 100]$ as a wide numerical domain (Remark 6).

Table 4 and Figure 3 show that exact rejection can coexist with small departures from the restrictions. Over three years the median industry requires a cost-efficiency allowance of 2.4%, and 79.8% of industries require less than 5%. Measurement error in factor prices

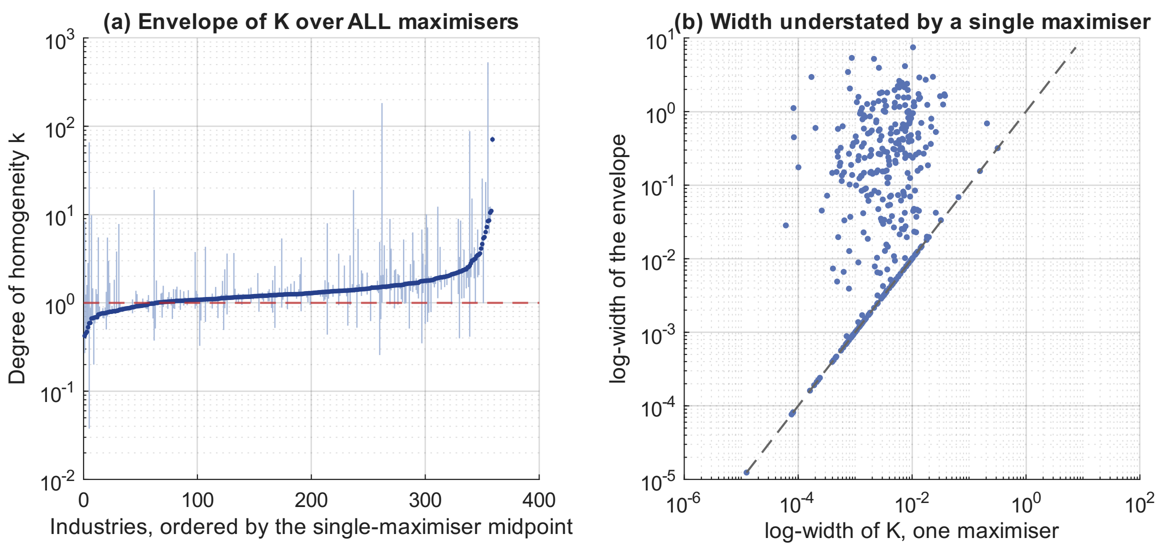

Figure 2: Non-uniqueness of maximum feasible subsets.

Notes: Panel (a): for each industry, the vertical bar is the envelope $[\min_T \underline{k},\ \max_T \overline{k}]$ of the identified set over *all* maximum-cardinality rationalizable subsets T; the dot is the geometric midpoint of $\mathcal{K}$ for one selected maximizer. Industries are ordered by that midpoint; the dashed line marks $k = 1$. Panel (b): log-width of the envelope against the log-width from a single maximizer, with the 45-degree line. Baseline specification, $r = 8\%$.

and quantities is one possible source of such departures (Varian, 1985). Over a decade the median requirement rises to 13.5%, and over the full panel to 27.7%. These are large model-relative allowances, but the deterministic exercise does not identify how much is due to measurement error rather than technology change, aggregation, or other departures from the model assumptions.

Two findings follow. First, the binary test in Table 1 does not convey how small most departures are over 2014–2016. Second, the required allowance rises substantially with the horizon. Monotonicity itself is partly mechanical: Proposition 3 shows that e^* cannot increase when observations are added. The empirical finding is the size of the decline; its direction follows from the construction of the index. Distinguishing technological drift from stationary measurement error would require a fixed-cardinality comparison or an explicit measurement-error model, neither of which is imposed here.

4.6 Dynamics and cross-industry heterogeneity

We use the consecutive-window results to examine changes in model fit over time and differences across industries.

Figure 4(a) plots the share of industries whose two-year window beginning in year τ

Table 4: Cost-efficiency index $e^*(S_i;\mathbb{K})$, by sample horizon

Sample	Years per industry	$e^*(S_i;\mathbb{K})$ p10	Median	p90	Min	Required slack
2014–2016 short panel	3	0.925	0.976	0.996	0.696	2.4%
2007–2016 decade	10	0.750	0.865	0.921	0.491	13.5%
1958–2016 full panel	59	0.532	0.722	0.822	0.176	27.7%

Notes: $e^*(S_i;\mathbb{K})$ is the largest $e \leq 1$ such that industry i's data satisfy the e-relaxed cost-minimization inequalities (5) for a single common $k \in \mathbb{K}$ (Definition 3), computed exactly by the concave program of Proposition 3 with $\mathbb{K} = [0.01, 100]$. Distributions are across the 361 industries. The "Required slack" column is computed from the unrounded median e^*, so it can differ by 0.1 percentage point from one minus the displayed three-decimal median. In the short panel, 79.8% of industries have $e^* \geq 0.95$ and 21.6% have $e^* \geq 0.99$; in the full panel, none reach 0.95. Baseline specification, $r = 8\%$.

passes. The series falls from about 0.80 at the start of the sample to below 0.45 at the end, reaching its minimum of 0.393 in 2009 and its maximum of 0.806 in 1958. A descriptive linear trend gives -3.79 percentage points per decade (robust standard error 0.50, $t = -7.61$); the corresponding trend for three-year windows is also negative ($t = -2.97$). Because adjacent annual windows overlap, formal inference on these trend coefficients should use a serial-correlation-robust procedure (for example, HAC standard errors); the reported t-statistics should otherwise be read descriptively. The cross-industry median identified degree, Figure 4(b), likewise trends down, by -0.036 per decade ($t = -4.16$) from a level of 1.11. This is a shift in the cross-sectional median among passing windows, not a within-industry estimate of technological drift, because the composition of passing industries can change over time.

The low pass rate in 2009 raises the question of whether the pattern is related to business-cycle changes in capacity utilization. Using the coarse indicator of whether a window begins in an NBER-dated recession year, the mean pass rate is 0.618, against 0.619 in expansion years, a difference of -0.001 ($t = -0.04$, $p = 0.97$). This near-zero difference shows that pass rates are similar under this annual recession classification; it does not rule out cyclical utilization effects measured in other ways.

The pairwise restriction compares an observed bundle with a radially rescaled bundle from an adjacent year, so it should bind when output moves relative to the measured input bundle. Table 5 examines this association using all 20,938 two-year windows, regressing the pass indicator on absolute log changes in output, in a Divisia input aggregate, and in their difference, absorbing industry fixed effects. The coefficient on the output-input gap is negative (-2.61, $t = -21.6$), while the coefficient on input growth is close to zero ($t = 0.39$). Pass rates fall from 0.730 in the bottom quartile of the gap to 0.475 in the top quartile.

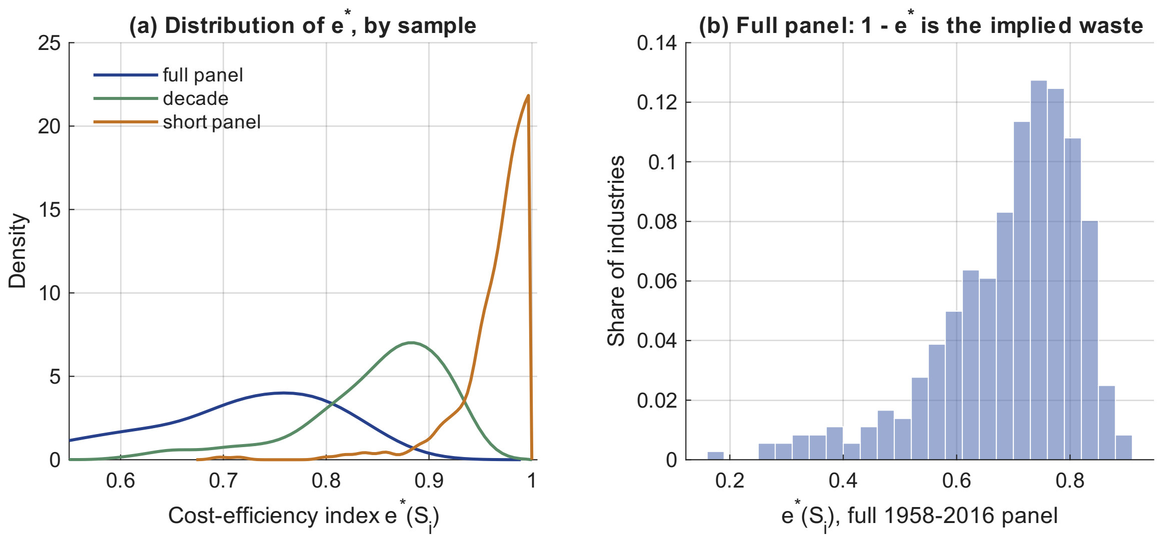

Figure 3: Distribution of the cost-efficiency index $e^*(S_i; \mathbb{K})$ across the 361 industries.

Notes: Panel (a): kernel densities for the three sample horizons of Table 4. Panel (b): histogram for the full 1958–2016 panel; $1 - e^*$ is the required proportional cost-efficiency slack under the maintained model, not literal waste. $\mathbb{K} = [0.01, 100]$; baseline specification, $r = 8\%$.

This diagnostic pattern is consistent with the channel emphasized by the framework, but it is not a causal mechanism test because the output–input gap is constructed from the same movements that enter the revealed-preference inequalities. It also aligns with the utilization-based reading of measured returns to scale (Basu, 1996; Basu et al., 2006; Burnside et al., 1995): the restrictions are less often satisfied when output changes differ more from changes in measured inputs. It is also consistent with the sparse-versus-local pattern documented in Section 4.3, without by itself establishing a causal comparison between adjacent and distant years.

Table 6 reports the cross-industry heterogeneity exercises. It relates industry-level consistency to observable characteristics computed from the same data. The regression for the efficiency index has the highest explanatory power, with $R^2 = 0.33$: consistency is significantly lower in industries with faster capital deepening (-2.91, $t = -5.11$), more volatile output growth (-0.68, $t = -4.31$), a larger capital share (-1.30, $t = -3.94$) or a larger materials share (-0.44, $t = -5.96$), and significantly higher in larger industries ($t = 4.07$). The model fits less well in industries with a large capital share or rapid capital deepening. Capital is also the input whose price must be imputed and whose service flow is least well measured by a perpetual-inventory stock, so the correlations do not distinguish genuine technology change from capital mismeasurement.

The regressions in the other two columns explain less of the variation across industries.

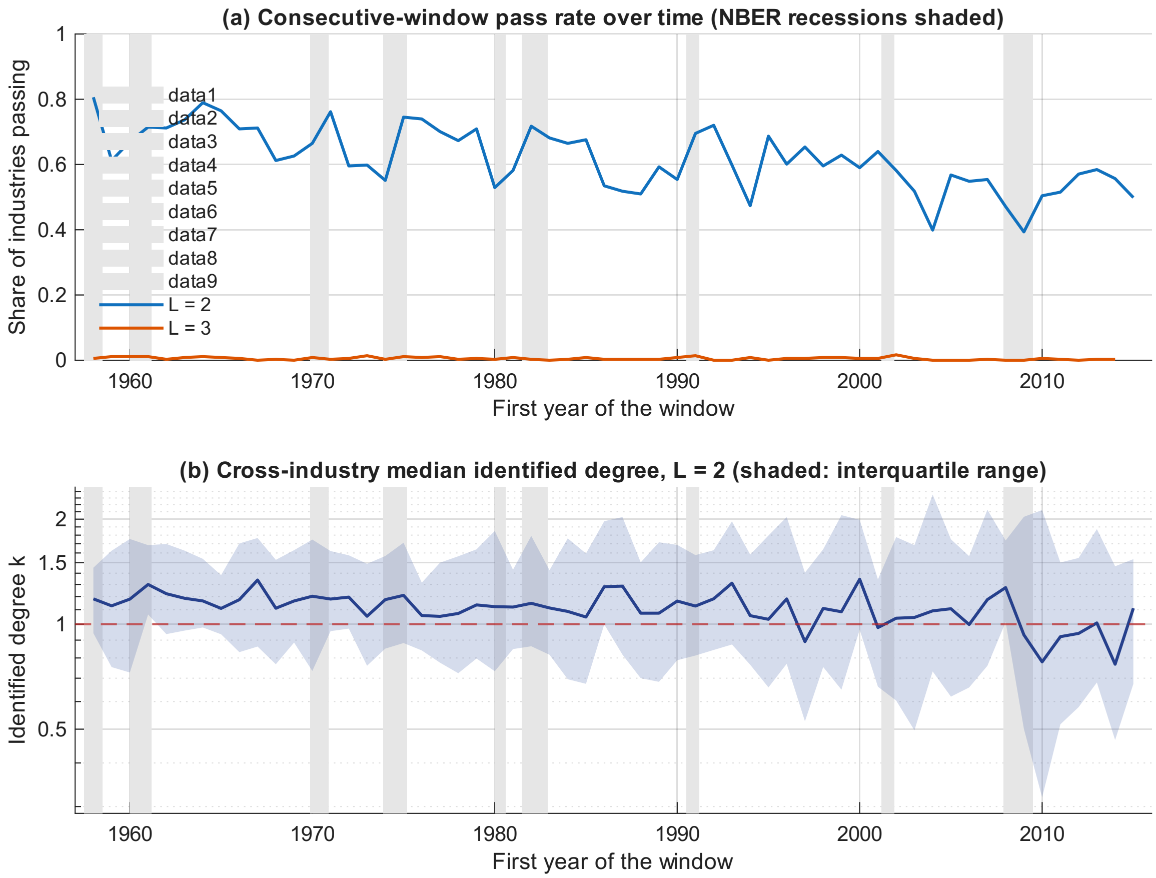

Figure 4: Dynamics of consecutive-window rationalizability.

Notes: Panel (a): share of the 361 industries whose window beginning in year τ has $\mathcal{K}(S_i) \neq \varnothing$, for $L = 2$ and $L = 3$. Panel (b): cross-industry median identified degree among passing two-year windows, with the interquartile range shaded; the dashed line marks $k = 1$. NBER-dated recessions shaded in grey. Baseline specification, $r = 8\%$.

$L_i^{\max}$ takes only two values in these data, so its cross-industry variance is small by construction and the fit is correspondingly poor ($R^2 = 0.07$); the identified degree is likewise only weakly predicted ($R^2 = 0.05$). All three columns should be interpreted descriptively: the regressors are outcomes of the same industry dynamics that generate the dependent variable, so the coefficients are correlations, not causal effects.

4.7 Robustness

This section presents a robustness analysis examining the sensitivity of the results to the chosen interest rate and the capital utilization.

Table 5: What predicts failure of a two-year window

	Coefficient	Robust s.e.	t
$\lvert\Delta \log y\rvert$	1.207	0.095	12.65
$\lvert\Delta \log x\rvert$ (Divisia aggregate)	0.039	0.102	0.39
$\lvert\Delta \log y - \Delta \log x\rvert$	−2.614	0.121	−21.62
Industry fixed effects		Yes	
Observations		20,938	
Mean of dependent variable		0.619	
Memo: pass rate by quartile of $\lvert\Delta \log y - \Delta \log x\rvert$: bottom quartile 0.730 top quartile 0.475			

Notes: Linear probability model. The dependent variable is one if the two-year window has $\mathcal{K}(S_i) \neq \varnothing$. $\Delta \log x$ is the expenditure-share-weighted (Divisia) log change in the five-input bundle, so $\Delta \log y - \Delta \log x$ is a Solow-residual-style measure of output movement not accounted for by measured inputs. The reported standard errors are heteroskedasticity-robust. Because adjacent windows overlap within industries and common annual shocks can induce cross-industry dependence, the reported t-statistics are descriptive. Formal inference would require an appropriate adjustment for these dependencies, such as two-way clustering. Baseline specification, $r = 8\%$.

The required rate of return. The user cost (9) requires a rate r that the NBER-CES data do not identify. Table 7 repeats the key tests at $r \in \{6\%, 8\%, 10\%\}$. The full-panel and leave-one-year-out results are identical at all three values. Pass rates also fall sharply with window length at all three values: two-year pass rates lie between 0.616 and 0.621, three-year pass rates between 0.005 and 0.006, four-year passes number zero, one, or two out of 20,216, and five-year and longer passes number exactly zero. The 2014–2016 short-panel count ranges from one to two industries.

Excluding capital altogether. We also exclude capital from the priced input vector and use a four-input short-run specification $x_{it} = (L_{it}, N_{it}, M_{it}, E_{it})$ treating capital as quasi-fixed. This is only as credible as the premise that capital is roughly constant across the compared observations, so we report alongside it the ratio $\max_t K_{it}/ \min_t K_{it}$ over the retained years. Over 2014–2016 that ratio has mean 1.06 and median 1.04, so the premise is reasonable, and excluding capital yields 2 nonempty industries (233 and 330) against 1 (industry 78) in the baseline. The passing industries differ across the two specifications, so their identity at short horizons depends on the input set. Over the full panel the ratio has mean 8.46 and median 4.14, reaching 184 for one industry, so the quasi-fixed assumption is not plausible over the full panel. The four-input result (0 nonempty, as in the baseline) therefore provides only limited additional evidence. Detailed industry-level results under this specification are in Appendix C.

5 Discussion

5.1 What the rejection does and does not establish

The test jointly assumes a time-invariant homogeneous technology, exact cost minimization at observed prices, and correctly measured inputs, output, and prices. A rejection shows that these assumptions cannot all hold for the observed data. It does not identify which assumption fails. Technical change, factor-biased innovation, variation in capacity utilization, aggregation across heterogeneous establishments within a six-digit industry, changing product mix, input-market power or other wedges between observed factor prices and marginal acquisition costs, and measurement error in factor prices and quantities can each generate the observed violations.

Three findings help describe these departures from the model.

First, the failure is *graded and becomes more severe as the horizon expands* (Table 4). Median required cost-efficiency slack rises from 2.4% over three years to 13.5% over ten years and 27.7% over fifty-nine years. Part of this deterioration is mechanical because adding observations adds inequalities and Proposition 3 forces e^* weakly downward. Its magnitude is nevertheless economically informative about how difficult the joint maintained hypothesis becomes over longer samples. We do not use this pattern alone to distinguish technological drift from measurement error or aggregation.

Second, the failure is *concentrated on comparisons where output moves relative to measured inputs* (Table 5). The output-input gap, a measure similar to the Solow residual, is negatively associated with passing the test ($t = -21.6$), while the coefficient on input movement alone is close to zero. This pattern is consistent with the utilization-based interpretation in (Basu, 1996; Burnside et al., 1995; Basu et al., 2006), under which measured inputs do not capture the service flow actually used. Because the output–input gap is built from the same variables entering the revealed-preference inequalities, however, we treat it as a diagnostic correlation rather than independent identification of that mechanism.

Third, the failure is *larger in industries with a high capital share or rapid capital deepening* (Table 6). Capital deepening and the capital share both predict lower e^* significantly. The price of capital must be imputed, and a perpetual-inventory stock may poorly measure its service flow, so this is consistent both with genuine capital-embodied technical change and with capital mismeasurement. The data do not allow us to distinguish these explanations.

The rejection persists across the capital-price specifications considered and when capital is excluded from the priced input vector (Table 7), subject to the limitations of the quasi-fixed-capital assumption discussed above.

5.2 Sparse versus local rationalizability

Section 4.3 distinguishes the number of rationalizable observations from the length of a rationalizable period. Maximum-feasible-subset statistics of the Houtman and Maks (1985) type are widely used to summarize how badly a revealed-preference model fails, and they are informative for that purpose. They do not describe *which* observations are retained, which matters in panel data. Here the median industry retains five years, but no industry can rationalize five consecutive years under any specification (or four, in the baseline), and the retained years span a median of 43 calendar years with a median adjacent run of one.

The implication for practice is direct. The joint restrictions of a stable homogeneous technology and exact cost minimization are rejected over every decade-long window in this industry panel. No choice of a five-year-or-longer consecutive subwindow within such a decade can rescue that assumption, since no five-year consecutive block survives anywhere in the panel; sparse nonconsecutive subsets are a different exercise. By contrast, the same researcher working with two-year differences finds greater support for the model: 62% of adjacent pairs pass exactly, and the median industry needs only a few percent of slack over three years. The results are substantially more compatible with short-horizon than with long-horizon common-technology specifications. They therefore caution against imposing long-horizon stability without an explicit allowance for technology change, aggregation, or measurement error.

5.3 Returns to scale as a set-identified object

Where the model does survive, returns to scale are recovered rather than assumed. The identified set $\mathcal{K}(S)$ collects every degree consistent with observed cost-minimizing choices; it is a singleton only when the tightest lower and upper bounds coincide, and otherwise an interval whose width measures how informative the data are. Among passing two-year windows the median identified degree is 1.13 with an interquartile range of $[0.77, 1.64]$, so even where the model holds, the data do not generally determine returns to scale precisely, and both increasing (59%) and decreasing (40%) returns are common.

Using one selected subset per industry suggests predominantly increasing returns, since 82% of selected maximal subsets imply IRS. Section 4.4 shows that figure is not selection-invariant: requiring agreement across all maximizers leaves 66% IRS, 10% DRS, and 24% ambiguous. Taken together, the results show that identified degrees cluster modestly above one, that constant returns is admissible for very few industries under any selection, and that the cross-industry distribution of k is not precisely estimated by this method.

5.4 Limitations

The analysis has four main limitations.

No sampling theory. The analysis is deterministic: data are treated as observed without error, and every statement is a statement about the observed dataset. There are no standard errors on "$\mathcal{K}(S_i) = \varnothing$", and the reported t-statistics in Tables 5 and 6 treat the computed indices as data, not as estimates. An extension to formal inference could provide confidence sets for $\mathcal{K}(S)$ under sampling or measurement error, in the spirit of Chernozhukov et al. (2007) for set-identified parameters, Varian (1985) for measurement error in revealed-preference tests, or the bootstrap RTS testing framework of Simar and Wilson (2002). This would allow researchers to assess whether the e^* values in Table 4 are statistically distinguishable from one. We do not take it up here.

Aggregation. The unit is a six-digit industry, not a plant. Industry aggregates of cost-minimizing plants need not themselves satisfy the restrictions, so part of the rejection is plausibly aggregation. Plant-level data with input prices would help assess the role of aggregation; the NBER-CES data cannot.

Cost minimization, not profit maximization. The test assumes prices are taken as given and that observed input bundles minimize the cost of the observed output. Markups alone do not invalidate this assumption, since profit maximization implies cost minimization in the static price-taking setting. Input-market power, adjustment costs, and dynamic factor demand can invalidate it, and capital is precisely the input for which static cost minimization is least plausible. The four-input specification is a partial response, and its own premise fails over long horizons.

The capital price. No rental price of capital is observed, and the user cost (9) requires a rate of return the data do not identify. The main rejection patterns persist across $r \in \{6\%, 10\%\}$ and when capital is excluded, but the underlying capital *quantity* is a perpetual-inventory stock, not a service flow, and changing r does not address that measurement issue.

Despite these limitations, the findings provide a useful basis for assessing homogeneous production models in empirical work. The framework identifies which degrees of homogeneity are consistent with observed choices without specifying a parametric production function, and measures the relaxation needed when the exact restrictions fail. In the manufacturing data, the contrast between small departures over short horizons and larger departures over longer periods shows why the observation horizon matters for model fit. The distinction between consecutive windows and scattered subsets, combined with differences across equally large rationalizable subsets, also shows why the number of retained observations alone cannot establish technological stability or a unique returns-to-scale classification. These findings help researchers assess the support for a common technology and make explicit the qualifications needed when interpreting returns to scale.

6 Conclusion

A common degree of returns to scale is informative only to the extent that the observations support the restrictions under which it is recovered. This paper provides a nonparametric framework for assessing those restrictions. Building on Varian's constant-returns test, it characterizes the complete set of homogeneity degrees consistent with observed-price cost minimization. The pairwise structure yields closed-form bounds and short rejection certificates. When the identified set is empty, cost-efficiency allowances and observation deletions measure different dimensions of the departure from the model.

The manufacturing evidence illustrates the importance of assessing fit alongside identification. Exact rationalizability disappears rapidly as consecutive observations are added, even though a different common degree may be chosen for each window. Selecting a larger rationalizable subset does not recover a corresponding period of stability: the selected years are typically separated by decades. Alternative maximum-cardinality subsets can also support different returns-to-scale classifications. A classification obtained from a selected subset therefore need not summarize the evidence in the industry panel. At the same time, the efficiency results show that exact rejection can coexist with a small required relaxation over short horizons. The size of the required relaxation therefore helps interpret exact rejection.

The interpretation remains conditional on the maintained model. The tests jointly impose a time-invariant homogeneous technology, exact cost minimization, and correctly measured quantities and prices. They do not separate technological change from utilization, adjustment frictions, or measurement error. The cost-efficiency allowance quantifies the relaxation of the revealed restrictions and should not be interpreted as measured productive waste. Accordingly, the findings provide evidence about the adequacy of a common homogeneous representation, rather than identifying the mechanism behind its failure.

For empirical work, the framework offers a way to report what the data support before selecting a production function or a single scale parameter: the admissible degrees, the comparisons that rule them out, and the sensitivity of fit to the observation horizon. Extending these diagnostics to accommodate explicit measurement error, productivity heterogeneity, and sampling uncertainty would help determine which relaxations restore a useful common representation while preserving informative restrictions on returns to scale.

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A Proofs

Proof of Theorem 1

Proof. **1 $\Rightarrow$ 2.** Suppose f is monotone, homogeneous of degree k, and cost-rationalizes S, so that $x_t \in \arg\min\{w_t \cdot x : f(x) \geq y_t\}$ for each t. Fix s, t and set $\lambda = (y_t/y_s)^{1/k}$. Homogeneity gives $f(\lambda x_s) = \lambda^k f(x_s) = (y_t/y_s)\, y_s = y_t$, so λx_s is feasible for output y_t. Cost minimization at t then gives $w_t \cdot x_t \leq \lambda\,(w_t \cdot x_s)$. The symmetric argument, exchanging the roles of s and t and using $\mu = (y_s/y_t)^{1/k} = 1/\lambda$, gives $w_s \cdot x_s \leq \mu\,(w_s \cdot x_t)$, that is, $v_s \cdot x_t \geq 1/\mu = (y_t/y_s)^{1/k}$, which is (1).

2 $\Rightarrow$ 3. Define $\tilde{h}(x) = \min_{t \in \mathbb{T}} y_t(v_t \cdot x)^k$. Each term is continuous and nondecreasing in x (as $v_t \gg 0$), so $\tilde{h}$ is continuous and monotone; and $\tilde{h}(\lambda x) = \lambda^k \tilde{h}(x)$, so $\tilde{h}$ is homogeneous of degree k.

Quasi-concavity. Each map $x \mapsto y_t(v_t \cdot x)^k$ is the composition of the strictly increasing function $z \mapsto y_t z^k$ on $\mathbb{R}_+$ with the linear map $x \mapsto v_t \cdot x$. A monotone transformation of a linear function is quasi-concave, so each term is quasi-concave; and the pointwise minimum of quasi-concave functions is quasi-concave, because for any level c the upper contour set $\{x : \tilde{h}(x) \geq c\} = \bigcap_t \{x : y_t(v_t \cdot x)^k \geq c\} = \bigcap_t \{x : v_t \cdot x \geq (c/y_t)^{1/k}\}$ is an intersection of halfspaces, hence convex. Note this argument holds for *every* $k > 0$, including $k > 1$, and exhibits the upper contour sets explicitly as polyhedra.

Rationalization. Fix t and set $y_t^* = \tilde{h}(x_t)$. The t-th term gives $y_t^* \leq y_t(v_t \cdot x_t)^k = y_t$. Conversely, for any s, (1) gives $v_s \cdot x_t \geq (y_t/y_s)^{1/k}$, hence $y_s(v_s \cdot x_t)^k \geq y_t$; taking the minimum over s gives $y_t^* \geq y_t$. Therefore $\tilde{h}(x_t) = y_t$. Next, if $\tilde{h}(x) \geq y_t$ then in particular the t-th term satisfies $y_t(v_t \cdot x)^k \geq y_t$, so $v_t \cdot x \geq 1 = v_t \cdot x_t$ and hence $w_t \cdot x \geq w_t \cdot x_t$. Thus x_t minimizes cost for output y_t, and $\tilde{h}$ cost-rationalizes S.

3 $\Rightarrow$ 1. Immediate, since 3 is a strengthening of 1.

Concavity if and only if $k \leq 1$. For $k \in (0, 1]$ each term $y_t(v_t \cdot x)^k$ is the composition of the concave increasing function $z \mapsto y_t z^k$ with a linear function, hence concave, and the pointwise minimum of concave functions is concave. For $k > 1$, Remark 4 shows no monotone concave degree-k homogeneous function with $f(\mathbf{0}) = 0$ exists other than $f \equiv 0$, which cannot rationalize S since $y_t > 0$; in particular $\tilde{h}$, which satisfies $\tilde{h}(\mathbf{0}) = 0$ and $\tilde{h}(x_t) = y_t > 0$, is not concave. $\square$

Proof of Proposition 1

Proof. Let f be any monotone, degree-k homogeneous rationalizer, and fix $t \in \mathbb{T}$ and $x \in \mathbb{R}^L_+$ with $v_t \cdot x > 0$. Define $z := x/(v_t \cdot x)$, so that $v_t \cdot z = 1$ and hence $w_t \cdot z = w_t \cdot x_t$.

We claim $f(z) \leq y_t$. Suppose not, so $f(z) > y_t$. Put $\alpha := (y_t/f(z))^{1/k} \in (0,1)$. Homogeneity gives $f(\alpha z) = \alpha^k f(z) = y_t$, so αz produces y_t, while $w_t \cdot (\alpha z) = \alpha\, w_t \cdot x_t < w_t \cdot x_t$, contradicting the cost minimality of x_t for output y_t. Hence $f(z) \leq y_t$.

Using homogeneity again, $f(x) = f\big((v_t \cdot x)z\big) = (v_t \cdot x)^k f(z) \leq (v_t \cdot x)^k y_t$. Since t was arbitrary, $f(x) \leq \min_t y_t (v_t \cdot x)^k = \tilde{h}(x)$. If $v_t \cdot x = 0$ for some t, then $v_t \gg 0$ and $x \geq 0$ imply $x = \mathbf{0}$. Homogeneity gives $f(\mathbf{0}) = 0$, and likewise $\tilde{h}(\mathbf{0}) = 0$, so the inequality holds. □

Proof of Proposition 2

Proof. Fix an ordered pair (s,t) and consider $a_{st} \geq q_{st}^{1/k}$ as a condition on $k > 0$. There are five exhaustive cases.

(i) $q_{st} > 1$, $a_{st} > 1$. Then $k \mapsto q_{st}^{1/k}$ is strictly decreasing, tending to 1 as $k \to \infty$ and to $+\infty$ as $k \to 0^+$. Taking logarithms, the condition is $\ln a_{st} \geq (1/k) \ln q_{st}$, that is $k \geq \ln q_{st} / \ln a_{st} > 0$: a lower bound.

(ii) $q_{st} < 1$, $a_{st} < 1$. Then $k \mapsto q_{st}^{1/k}$ is strictly increasing, tending to 1 as $k \to \infty$ and to 0 as $k \to 0^+$. Multiplying the log inequality by $k > 0$ and then dividing by $\ln a_{st} < 0$ reverses the inequality, giving $k \leq \ln q_{st} / \ln a_{st}$, a positive number: an upper bound.

(iii) $q_{st} = 1$. Then $q_{st}^{1/k} = 1$ for every k, so the condition reduces to $a_{st} \geq 1$, which holds for all k if true and for no k if false.

(iv) $q_{st} > 1$, $a_{st} \leq 1$. Then $q_{st}^{1/k} > 1 \geq a_{st}$ for every $k > 0$: the condition fails for all k.

(v) $q_{st} < 1$, $a_{st} \geq 1$. Then $q_{st}^{1/k} < 1 \leq a_{st}$ for every $k > 0$: no restriction.

Cases (iii, false) and (iv) are exactly the two stated impossibility conditions, and they exhaust the pairs for which the restriction fails for every $k > 0$. Note in particular that $a_{st} = 1$ with $q_{st} < 1$ falls under (v) and imposes no restriction; it is not a degenerate case requiring separate treatment. Away from the impossible pairs, each $\mathcal{K}_{st}$ is either $(0,\infty)$, a lower-bound interval $[\ell_{st}, \infty)$, or an upper-bound interval $(0, u_{st}]$. Their intersection is $(0,\infty) \cap [\underline{k}(S), \overline{k}(S)]$ with $\underline{k}(S)$ the largest lower bound and $\overline{k}(S)$ the smallest upper bound, as stated; the conventions $\max \varnothing = 0$ and $\min \varnothing = +\infty$ handle the cases in which no pair of the relevant type exists. Nonemptiness is then $\underline{k}(S) \leq \overline{k}(S)$, and point identification is equality. □

Proof of Theorem 2

Proof. By the case analysis in the previous proof, each $\mathcal{K}_{st}$ is either $\varnothing$, all of $(0, \infty)$, or a one-sided interval; in every case it is a convex subset of $(0, \infty)$.

If condition 1 or 2 holds, $\mathcal{K}(S) = \bigcap_{s,t} \mathcal{K}_{st}$ is trivially empty.

Conversely, suppose $\mathcal{K}(S) = \varnothing$ and no single $\mathcal{K}_{st}$ is empty. Among the nontrivial restrictions let $\underline{k}(S)$ be the largest lower endpoint of a lower-bound interval and $\overline{k}(S)$ the smallest upper endpoint of an upper-bound interval, with the conventions $\underline{k}(S) = 0$ and $\overline{k}(S) = +\infty$ if no interval of the relevant type is present. If either type is absent, the intersection equals the other one-sided interval intersected with $(0, \infty)$, which is nonempty, a contradiction. Thus both types are present, and $\bigcap_{s,t} \mathcal{K}_{st} = (0, \infty) \cap [\underline{k}(S), \overline{k}(S)] = \varnothing$ forces $\underline{k}(S) > \overline{k}(S)$. The two pairs attaining these extremes then satisfy $\mathcal{K}_{st} \cap \mathcal{K}_{pq} = \varnothing$, establishing condition 2.

(The sparsity is the one-dimensional Helly property for convex sets: a finite family of intervals on the line has empty intersection only if two of them already fail to intersect. What is specific here is the explicit description of each $\mathcal{K}_{st}$ in terms of q_{st} and a_{st}, which makes the certificate constructive and locatable in $O(T^2)$ time.) □

Proof of Proposition 3

Proof. Write $a_{st} = v_s \cdot x_t > 0$ and $q_{st} = y_t / y_s > 0$, and set $\theta := 1/k$, so $\Theta := [1/k_{\max}, 1/k_{\min}]$ is a compact subinterval of $(0, \infty)$. Define

$$\phi(\theta) := \min_{s,t \in \mathbb{T}} \bigl(\log a_{st} - \theta \log q_{st}\bigr).$$

Because $v_t \cdot x_t = 1$ and $y_t / y_t = 1$, the diagonal pairs contribute the constant 0 to the minimum, so $\phi(\theta) \le 0$ for every θ.

1. ϕ is a minimum of finitely many affine functions of θ, hence continuous (indeed concave and piecewise linear), and Θ is compact, so $\max_\Theta \phi$ is attained. Since $\phi > -\infty$ everywhere on Θ (each $a_{st} > 0$ and $q_{st} > 0$), $e^* = \exp(\max_\Theta \phi) \in (0, 1]$, the upper bound following from $\phi \le 0$.

2. $e^* = 1$ iff $\max_\Theta \phi = 0$ iff there is $\theta \in \Theta$ with $\log a_{st} \ge \theta \log q_{st}$ for every pair, iff there is $k = 1/\theta \in \mathbb{K}$ with $a_{st} \ge q_{st}^{1/k}$ for every pair, iff $\mathcal{K}(S) \cap \mathbb{K} \ne \varnothing$.

3. If $S \subseteq S'$ then the minimum defining $\phi_{S'}$ is over a superset of pairs, so $\phi_{S'} \le \phi_S$ pointwise on Θ; maximizing both sides preserves the inequality.

4. Replacing x_t^j by $c_j x_t^j$ and w_t^j by w_t^j / c_j leaves $w_t \cdot x_t$ and hence every $a_{st} = \sum_j (w_s^j x_s^j / w_s \cdot x_s)(x_t^j / x_s^j)$ unchanged; replacing y_t by $c y_t$ leaves every q_{st} unchanged; replacing w_t by $\gamma_t w_t$

leaves v_t unchanged. So ϕ, and hence e^*, is invariant.

5. ϕ is a pointwise minimum of affine functions of θ, hence concave; it is piecewise linear with finitely many pieces. Maximizing a concave function over an interval is the stated one-dimensional concave program, equivalent to the linear program $\max\{z : z \leq \log a_{st} - \theta \log q_{st}\ \forall s,t,\ \theta \in \Theta\}$ in the two variables (z, θ). Its optimum is global, and is characterized exactly by the linear program and can be approximated to arbitrary precision by golden-section search. □

Proof of Observations 1, 2, and 3

Proof. Observation 1. Any $k \in \mathcal{K}(S')$ satisfies (1) for all pairs in S', hence for all pairs in $S \subseteq S'$, so $k \in \mathcal{K}(S)$. If $\mathcal{K}(S) = \varnothing$ then $\mathcal{K}(S') = \varnothing$ too. Otherwise, since $\mathcal{K}(S') \subseteq \mathcal{K}(S)$, the infimum of $|k-1|$ over the smaller set is weakly larger, giving $\rho(S') \geq \rho(S)$.

Observation 2. Apply Observation 1 to $B \subseteq A$: $\mathcal{K}(S_A) \subseteq \mathcal{K}(S_B)$, so $\mathcal{K}(S_A) \neq \varnothing$ implies $\mathcal{K}(S_B) \neq \varnothing$. The second statement is the contrapositive.

Observation 3. With two observations the diagonal restrictions are vacuous ($a_{tt} = q_{tt} = 1$), leaving the two cross-observation inequalities. Since $y_1 \neq y_2$, exactly one of $q_{12} = y_2/y_1$ and $q_{21} = y_1/y_2$ exceeds one and the other is below one. By the case analysis in the proof of Proposition 2, the pair with ratio above one contributes either a lower bound (case i) or immediate emptiness (case iv), and the pair with ratio below one contributes either an upper bound (case ii) or no restriction (case v). Hence $\mathcal{K}(S)$ is either empty or an interval determined by at most one bound of each type, and is a singleton exactly when those bounds coincide. □

B Data appendix

B.1 Construction of output and inputs

We use the 2012 NAICS version of the NBER-CES Manufacturing Industry Database (Becker et al., 2021). The public database covers 1958–2018; we use 1958–2016 because the real capital stock needed for the five-factor specification is available through 2016. We drop three industries with problematic definitions in the construction file (NAICS 311811, 326212, 339116), leaving a balanced panel of 361 industries observed for 59 years.

Output is real gross output,

$$y_{it} = \frac{\mathrm{VSHIP}_{it}}{\mathrm{PISHIP}_{it}},$$

where VSHIP is the nominal value of shipments and PISHIP the shipments deflator. Gross output is preferred to value added because the revealed-preference inequalities compare the cost of producing an observed output level using all measured inputs.

The input vector follows the five-factor NBER-CES productivity construction, $x_{it} = (K_{it}, L_{it}, N_{it}, M_{it}, E_{it})$, with

$$K_{it} = \mathrm{CAP}_{it}, \qquad L_{it} = \mathrm{PRODH}_{it}, \qquad N_{it} = \mathrm{EMP}_{it} - \mathrm{PRODE}_{it},$$

$$E_{it} = \frac{\mathrm{ENERGY}_{it}}{\mathrm{PIEN}_{it}}, \qquad M_{it} = \frac{\mathrm{MATCOST}_{it} - \mathrm{ENERGY}_{it}}{\mathrm{PIMAT}_{it}}.$$

Nominal energy expenditure is subtracted from nominal materials cost before deflating by PIMAT, so that $\mathrm{PIMAT}_{it} M_{it} + \mathrm{PIEN}_{it} E_{it} = \mathrm{MATCOST}_{it}$ exactly; this avoids double-counting energy as both a material and a separate input. The PIMAT series is the materials-cost deflator rather than a deflator constructed specifically for non-energy materials, so this decomposition should be understood as the standard empirical approximation used to separate energy from the broader materials category. Observations with nonpositive output, input quantities, or factor expenditures, or with missing values, are excluded; in the final panel none occur, so the panel is balanced.

B.2 Input prices and the user cost of capital

The test requires only normalized prices, recoverable from expenditures via (8). For variable inputs, expenditures are observed directly or constructed from observed categories:

$$e^L_{it} = \mathrm{PRODW}_{it}, \qquad e^N_{it} = \mathrm{PAY}_{it} - \mathrm{PRODW}_{it},$$

$$e^E_{it} = \mathrm{ENERGY}_{it}, \qquad e^M_{it} = \mathrm{MATCOST}_{it} - \mathrm{ENERGY}_{it}.$$

No rental price of capital is reported. Its construction affects the test because capital expenditure enters the denominator of every factor share. We use the Jorgensonian user cost (9) with $P_{I,it} = \mathrm{PIINV}_{it}$, the NBER-CES investment-goods deflator, and

- δ_i, an industry-specific depreciation rate backed out from the same perpetual-inventory identity NBER-CES itself uses to build CAP_{it}, namely $K_t = (1 - \delta_{t-1})K_{t-1} + I_t$ with $I_t = \mathrm{INVEST}_t/\mathrm{PIINV}_t$, solved in reverse as $\delta_{t-1} = 1 - (K_t - I_t)/K_{t-1}$ and averaged within industry after discarding per-year values outside $[0.01, 0.30]$;
- π_i^K, the industry's average log-growth rate of PIINV between its first and last available year, which smooths annual fluctuations;
- r, the required nominal rate of return, the only input not identified by the data. The baseline is $r = 8\%$, with $r \in \{6\%, 10\%\}$ as a sensitivity band, roughly spanning long-run corporate bond yields over 1958–2016.

The constructed user costs are positive throughout the sample. The implied δ_i has median 5.7% and ranges from 3.9% to 9.0% across the 361 industries, in line with standard depreciation estimates for manufacturing capital, and every industry has at least one usable per-year estimate, so no industry relies on a fallback value. The implied π_i^K has median 2.7% per year and ranges from 0.4% to 3.3%. A small positive floor on $w_{K,it}$, included in case the capital-gains term were ever to dominate $r + \delta_i$, does not bind for any industry-year at any of the three values of r.

The NBER-CES source data already contain the investment (INVEST) and investment-deflator (PIINV) series this construction requires, alongside the real capital stock CAP_{it} already used in the input vector, so no external series are needed beyond r itself.

B.3 Computation and replication

All tests reduce to the closed-form pairwise bounds of Proposition 2, computed in $\theta = 1/k$ so that every restriction is linear. For each industry the $T \times T$ matrices of pairwise bounds are formed once and reused across all subset and window queries, reducing computation time: the consecutive-window sweep evaluates more than 180,000 windows in under one second of computation, and the exact enumeration of every maximum-cardinality rationalizable subset for all 361 industries takes under thirty seconds.

Maximum feasible subsets are found by exact branch and bound, using the downward-closed structure of Observation 2 for pruning and a greedy graph-coloring bound; the enumeration of *all* maximizers is a depth-first search in increasing index order with the same pruning, and no enumeration cap binds for any industry. The efficiency index is computed by golden-section search on the concave objective of Proposition 35, to numerical tolerance and verified against the endpoints.

A validation script checks that (i) the vectorized pairwise bounds reproduce the identified sets of the original routine to within 2.5×10^{-14} relative error over more than two thousand random subsets, with no disagreement about feasibility; (ii) $e^* = 1$ exactly when $\mathcal{K}(S) \cap \mathbb{K} \neq \varnothing$, with no exceptions across all 361 industries; (iii) $e^* \in (0, 1]$ and is monotone under data expansion in every case tested; (iv) Example 2 is reproduced to twelve digits; and (v) the min-envelope (2) is quasi-concave numerically at $k \in \{0.5, 1, 1.5, 3\}$ over sixteen thousand random draws.

C Additional results

C.1 Forcing years beyond the maximum feasible subset

By Observation 1, $\mathcal{K}(S)$ shrinks monotonically as observations are added, and by construction no $(m^* + 1)$-year subset is feasible, so adding any year to a maximizer makes the identified set empty. To quantify how quickly this happens, define the *infeasibility gap*

$$g(T) := \max\big\{0,\ \underline{k}(S_i(T)) - \overline{k}(S_i(T))\big\},$$

Conditional on there being no directly impossible pair, $g(T) = 0$ if and only if the lower and upper bounds overlap. If $\mathcal{I}(S_i(T)) \neq \varnothing$, the dataset is infeasible regardless of $g(T)$, so directly impossible candidates must be flagged separately rather than treated as zero-gap feasible sets. Starting from a maximizer $T^{(m)}$, we add the remaining years one at a time, at each step choosing the year that leaves the gap smallest. The reported greedy calculation should therefore be interpreted conditional on the implementation separately screening $\mathcal{I}(S_i(T))$ at each step. This greedy rule need not find the minimum-gap superset at each size, but it traces one natural path from the maximizer to the full sample; by Observation 1 every such path ends at the same full-sample endpoint, and only the intermediate path differs.

Table C.1 reports this for the three industries attaining $m^* = 9$. All three follow a similar pattern: the gap stays small for a few additional years, then accelerates, with most of the increase concentrated late in the path. None approaches a nonempty identified set at the full sample.

C.2 Largest feasible year subsets by industry

Table C.2 reports, industry by industry, one maximum-cardinality rationalizable subset under the baseline specification, and Table C.3 the same under the four-input specification. As Section 4.4 documents, these subsets are usually not unique: the median industry has five maximizers and one has 311. The calendar years listed therefore represent one admissible selection, and the returns-to-scale column should be read together with Table 3.

Table C.2: Largest feasible year subset by industry, 1958–2016

Industry	$\lvert T_i^*\rvert$	Removed	$\mathcal{K}(S_i(T_i^*))$	Regime	Kept years T_i^*
1	4	55	$[3.1559, 3.1749]$	IRS	1958 1965 2002 2008
2	4	55	$[1.3919, 1.4002]$	IRS	1961 1985 2014 2015

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Table C.2: Largest feasible year subset by industry, 1958–2016 (continued)

Industry	$\lvert T_i^*\rvert$	Removed	$\mathcal{K}(S_i(T_i^*))$	Regime	Kept years T_i^*
3	4	55	$[7.1034, 7.3351]$	IRS	1973 1977 1981 2011
4	5	54	$[1.5730, 1.5873]$	IRS	1959 1971 1979 1991 1994
5	4	55	$[0.6745, 0.6796]$	DRS	1997 2003 2004 2005
6	6	53	$[1.6532, 1.6583]$	IRS	1958 1959 1969 2002 2006 2016
7	4	55	$[1.1686, 1.1793]$	IRS	1959 1969 1989 1998
8	4	55	$[1.3763, 1.3810]$	IRS	1965 1966 1989 2000
9	5	54	$[1.0665, 1.0673]$	IRS	1964 1966 1970 1980 1994
10	6	53	$[1.2529, 1.2571]$	IRS	1959 1970 1972 1988 1992 1996
11	5	54	$[0.8025, 0.8094]$	DRS	1958 1978 2000 2008 2011
12	6	53	$[1.9948, 2.0154]$	IRS	1958 1962 1966 1975 1982 2006
13	5	54	$[1.1246, 1.1343]$	IRS	1963 1975 1987 2005 2014
14	7	52	$[1.7183, 1.7437]$	IRS	1958 1964 1967 1973 1985 1988 2010
15	4	55	$[1.6498, 1.6675]$	IRS	1981 1986 2000 2016
16	5	54	$[1.3184, 1.3191]$	IRS	1962 1965 1989 1994 1998
17	6	53	$[2.3855, 2.4139]$	IRS	1958 1960 1972 1985 1997 2015
18	5	54	$[1.9114, 1.9257]$	IRS	1959 1961 1983 1995 2010
19	4	55	$[1.5929, 1.6081]$	IRS	1995 1997 2006 2016
20	5	54	$[10.6393, 11.3559]$	IRS	1958 1973 1977 1987 1991
21	4	55	$[0.9678, 0.9679]$	DRS	1962 1963 1972 2004
22	4	55	$[0.9120, 0.9174]$	DRS	1960 1963 1994 2001
23	5	54	$[0.8922, 0.8963]$	DRS	1958 1966 1967 1988 2009
24	4	55	$[2.0146, 2.0648]$	IRS	1966 1979 1997 2002
25	4	55	$[1.6861, 1.6917]$	IRS	1958 1969 1988 1992
26	4	55	$[1.2220, 1.2225]$	IRS	1961 1965 1984 1995
27	4	55	$[3.4173, 3.5391]$	IRS	1959 1962 1993 2008
28	4	55	$[1.4006, 1.4011]$	IRS	1973 1989 2000 2016
29	4	55	$[0.4228, 0.4231]$	DRS	1963 1974 1990 2003
30	5	54	$[1.2531, 1.2542]$	IRS	1975 1981 1985 2011 2013
31	4	55	$[1.0282, 1.0332]$	IRS	1958 1965 2008 2014
32	7	52	$[0.9783, 0.9798]$	DRS	1958 1975 1979 1988 1996 1998 2014
33	5	54	$[1.1801, 1.1855]$	IRS	1958 1980 1989 1996 1998
34	5	54	$[1.4092, 1.4118]$	IRS	1958 1969 1973 1991 2016
35	5	54	$[1.6366, 1.6393]$	IRS	1958 1965 1966 1975 1983
36	4	55	$[1.3209, 1.3378]$	IRS	1958 1965 1971 1998
37	3	56	$[1.4904, 1.4916]$	IRS	1962 1966 1973
38	4	55	$[2.7703, 2.8309]$	IRS	1966 1995 1999 2016
39	5	54	$[1.2983, 1.3051]$	IRS	1958 1968 1982 1994 2012
40	4	55	$[1.2473, 1.2527]$	IRS	1963 1968 1974 2001
41	5	54	$[1.1834, 1.1865]$	IRS	1963 1967 1971 1998 2010
42	5	54	$[1.4521, 1.4543]$	IRS	1958 1967 1971 1990 2016
43	6	53	$[0.9788, 0.9790]$	DRS	1960 1965 1972 1977 1996 2007
44	6	53	$[1.2183, 1.2246]$	IRS	1960 1966 1967 1977 1989 1993
45	5	54	$[1.2468, 1.2625]$	IRS	1959 1965 1977 1978 1984
46	5	54	$[3.5934, 3.6649]$	IRS	1959 1964 1965 1987 2016
47	4	55	$[1.2803, 1.2818]$	IRS	1958 1962 1995 2009
48	4	55	$[7.7248, 9.4960]$	IRS	1959 1974 1982 2015
49	5	54	$[1.0105, 1.0185]$	IRS	1976 1983 1988 1999 2011
50	6	53	$[1.1662, 1.1803]$	IRS	1963 1967 1971 1979 1992 2001

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Table C.2: Largest feasible year subset by industry, 1958–2016 (continued)

Industry	$\|T_i^*\|$	Removed	$\mathcal{K}(S_i(T_i^*))$	Regime	Kept years T_i^*
51	5	54	$[0.8889, 0.8896]$	DRS	1970 1977 1983 2008 2012
52	5	54	$[0.8720, 0.8802]$	DRS	1986 1998 2001 2006 2015
53	6	53	$[1.3048, 1.3071]$	IRS	1958 1967 1978 1989 2001 2002
54	5	54	$[1.4880, 1.4936]$	IRS	1958 1960 1982 1988 1994
55	5	54	$[0.8840, 0.9030]$	DRS	1959 1963 1981 1983 1987
56	5	54	$[1.3544, 1.3609]$	IRS	1958 1981 2000 2002 2009
57	6	53	$[1.3723, 1.3763]$	IRS	1959 1966 1972 1986 1987 2005
58	6	53	$[1.9406, 1.9586]$	IRS	1968 1975 1988 1994 1995 2005
59	5	54	$[1.5358, 1.5407]$	IRS	1962 1974 2010 2013 2016
60	5	54	$[1.9634, 1.9685]$	IRS	1960 1961 1994 1998 2000
61	5	54	$[1.5252, 1.5368]$	IRS	1958 1962 1974 1995 1999
62	6	53	$[0.9187, 0.9189]$	DRS	1983 1984 1993 2003 2011 2013
63	5	54	$[1.0700, 1.0842]$	IRS	1981 1982 2003 2010 2016
64	4	55	$[0.4650, 0.4737]$	DRS	1977 1981 2010 2014
65	5	54	$[0.9785, 0.9843]$	DRS	1960 1966 1976 2008 2012
66	5	54	$[0.8250, 0.8310]$	DRS	1984 1986 2001 2006 2016
67	4	55	$[0.5904, 0.5962]$	DRS	2000 2008 2013 2016
68	6	53	$[2.4976, 2.5166]$	IRS	1960 1965 1967 1968 1992 1995
69	4	55	$[1.7816, 1.7934]$	IRS	1958 1968 2001 2002
70	5	54	$[0.8409, 0.8422]$	DRS	1959 1971 2000 2008 2015
71	5	54	$[0.8055, 0.8071]$	DRS	1978 1991 2004 2015 2016
72	5	54	$[0.8411, 0.8422]$	DRS	1978 1989 1990 2007 2014
73	5	54	$[1.9334, 1.9522]$	IRS	1958 1960 1978 1981 2002
74	5	54	$[0.9999, 1.0008]$	CRS	1960 1989 1990 2006 2014
75	3	56	$[2.6282, 2.6305]$	IRS	1996 2003 2007
76	5	54	$[1.3528, 1.3626]$	IRS	1958 1969 1986 1988 2005
77	5	54	$[1.0746, 1.0754]$	IRS	1975 1981 1984 1990 2014
78	5	54	$[1.1984, 1.2011]$	IRS	1960 1963 1966 1974 2016
79	5	54	$[1.4219, 1.4404]$	IRS	1977 1986 1989 2000 2008
80	5	54	$[0.9219, 0.9247]$	DRS	1959 1966 1988 2003 2016
81	5	54	$[1.7843, 1.7896]$	IRS	1961 1963 1972 1986 2014
82	6	53	$[1.0235, 1.0244]$	IRS	1958 1959 1965 1990 2012 2016
83	5	54	$[1.0097, 1.0173]$	IRS	1996 1997 1999 2002 2015
84	4	55	$[1.2006, 1.2054]$	IRS	1960 1967 1998 2000
85	4	55	$[1.1332, 1.1348]$	IRS	1960 1966 1970 1981
86	5	54	$[1.4837, 1.5033]$	IRS	1959 1964 1997 2006 2016
87	5	54	$[1.9673, 2.0050]$	IRS	1960 1962 1971 1997 2001
88	5	54	$[1.4837, 1.4905]$	IRS	1961 1977 1982 1986 1998
89	5	54	$[0.6664, 0.6724]$	DRS	1979 1982 1999 2007 2013
90	6	53	$[5.5453, 5.5728]$	IRS	1970 1978 1979 1984 2005 2006
91	5	54	$[1.1454, 1.1527]$	IRS	1961 1967 1968 1987 1994
92	5	54	$[1.3822, 1.3855]$	IRS	1962 1965 1969 1990 1997
93	7	52	$[2.1475, 2.1511]$	IRS	1969 1970 1986 1992 1998 2003 2015
94	5	54	$[1.3045, 1.3059]$	IRS	1958 1962 1983 1985 2004
95	6	53	$[1.3586, 1.3596]$	IRS	1958 1975 1978 1995 2002 2006
96	5	54	$[1.2805, 1.2820]$	IRS	1962 1964 1965 1998 2002
97	5	54	$[1.5591, 1.5657]$	IRS	1975 1985 1995 1999 2006
98	5	54	$[1.0260, 1.0282]$	IRS	1975 1980 1985 1992 1999

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Table C.2: Largest feasible year subset by industry, 1958–2016 (continued)

Industry	$\lvert T_i^* \rvert$	Removed	$\mathcal{K}(S_i(T_i^*))$	Regime	Kept years T_i^*
99	4	55	$[2.2021, 2.2207]$	IRS	1965 1970 1987 2008
100	5	54	$[1.8880, 1.9029]$	IRS	1978 1980 1983 1987 2002
101	4	55	$[0.7352, 0.7407]$	DRS	1995 2000 2010 2016
102	4	55	$[1.1041, 1.1124]$	IRS	1964 1966 1972 2016
103	5	54	$[0.9766, 0.9836]$	DRS	1978 1979 1981 1982 1992
104	5	54	$[1.5230, 1.5292]$	IRS	1958 1972 1990 1992 2005
105	4	55	$[0.9947, 1.0016]$	CRS	1958 1963 1986 1997
106	4	55	$[1.7424, 1.7834]$	IRS	1958 1960 1971 2001
107	4	55	$[1.0998, 1.1013]$	IRS	1964 1966 1968 1987
108	5	54	$[2.2621, 2.2679]$	IRS	1960 1961 1973 2004 2013
109	5	54	$[1.8817, 1.8855]$	IRS	1958 1966 1967 1972 2002
110	6	53	$[1.8113, 1.8371]$	IRS	1962 1963 1973 1975 1991 2015
111	4	55	$[2.0548, 2.0630]$	IRS	1959 1960 1961 1964
112	5	54	$[2.0167, 2.0533]$	IRS	1959 1965 1970 1973 2004
113	4	55	$[1.1394, 1.1459]$	IRS	1964 1967 1993 1994
114	4	55	$[1.2802, 1.2888]$	IRS	1959 1978 1979 1987
115	5	54	$[1.3101, 1.3176]$	IRS	1958 1960 1974 2009 2016
116	4	55	$[1.6139, 1.6566]$	IRS	1958 1960 1985 1986
117	6	53	$[2.5382, 2.5522]$	IRS	1959 1962 1974 1996 1997 2001
118	4	55	$[1.5157, 1.5189]$	IRS	1958 1972 1980 1982
119	4	55	$[8.3913, 8.4545]$	IRS	1959 1987 1991 2010
120	5	54	$[1.3730, 1.3799]$	IRS	1959 1961 1981 1991 1995
121	5	54	$[1.4861, 1.4906]$	IRS	1969 1974 1975 1995 2001
122	5	54	$[0.7590, 0.7686]$	DRS	1976 1982 1991 1994 2008
123	5	54	$[1.3331, 1.3348]$	IRS	1964 1993 1996 2005 2006
124	5	54	$[1.3246, 1.3294]$	IRS	1966 1968 1974 1992 2004
125	5	54	$[1.5880, 1.5903]$	IRS	1959 1978 1992 1993 2005
126	5	54	$[1.1396, 1.1438]$	IRS	1961 1991 1997 2008 2013
127	4	55	$[1.5584, 1.5615]$	IRS	1962 1965 1970 1993
128	5	54	$[2.2811, 2.2865]$	IRS	1961 1967 1982 1994 2004
129	5	54	$[0.4486, 0.4526]$	DRS	1986 1994 2003 2014 2016
130	6	53	$[0.7589, 0.7599]$	DRS	1982 1987 1989 1990 2000 2009
131	5	54	$[1.0862, 1.0890]$	IRS	1975 1985 2001 2003 2012
132	4	55	$[1.2359, 1.2405]$	IRS	1963 1989 2002 2003
133	7	52	$[1.1914, 1.1930]$	IRS	1958 1964 1978 1995 1997 2004 2016
134	6	53	$[1.5051, 1.5125]$	IRS	1958 1961 1962 1988 2000 2004
135	5	54	$[1.0217, 1.0237]$	IRS	1986 1992 1995 1998 2015
136	4	55	$[1.2175, 1.2212]$	IRS	1958 1979 1995 2001
137	5	54	$[1.3749, 1.3801]$	IRS	1961 1976 1983 1985 1999
138	6	53	$[1.2274, 1.2281]$	IRS	1958 1960 1967 1990 1994 2011
139	5	54	$[1.2463, 1.2495]$	IRS	1958 1962 1977 1988 1991
140	5	54	$[1.1896, 1.1913]$	IRS	1958 1961 1962 1978 2000
141	5	54	$[1.2328, 1.2383]$	IRS	1958 1962 1965 1993 1997
142	5	54	$[1.2532, 1.2533]$	IRS	1960 1974 1983 1987 2009
143	6	53	$[1.2613, 1.2620]$	IRS	1958 1960 1991 1992 2003 2009
144	5	54	$[1.2885, 1.2921]$	IRS	1958 1960 1969 1972 2004
145	5	54	$[1.2916, 1.2921]$	IRS	1958 1976 1983 1987 1996
146	6	53	$[1.2980, 1.2983]$	IRS	1960 1975 1982 1985 1993 1998

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Table C.2: Largest feasible year subset by industry, 1958–2016 (continued)

Industry	$\lvert T_i^* \rvert$	Removed	$\mathcal{K}(S_i(T_i^*))$	Regime	Kept years T_i^*
147	4	55	$[4.6018, 4.6619]$	IRS	1958 1961 1996 2006
148	6	53	$[1.0985, 1.1038]$	IRS	1958 1960 1968 1977 2006 2010
149	5	54	$[1.7910, 1.7924]$	IRS	1958 1962 1979 1991 2004
150	5	54	$[1.5867, 1.5880]$	IRS	1958 1962 1965 1987 1991
151	5	54	$[0.8568, 0.8576]$	DRS	1973 1988 2002 2010 2012
152	3	56	$[0.5847, 0.5908]$	DRS	1974 1994 2009
153	5	54	$[6.3196, 6.3512]$	IRS	1960 1967 1969 1982 2005
154	5	54	$[1.2066, 1.2446]$	IRS	1958 1960 1973 1993 2001
155	5	54	$[1.4109, 1.4110]$	IRS	1958 1961 1963 1965 1991
156	5	54	$[1.2060, 1.2071]$	IRS	1980 1986 1990 1994 2002
157	7	52	$[4.1076, 4.1325]$	IRS	1959 1971 1982 1984 1988 2001 2010
158	6	53	$[1.3386, 1.3411]$	IRS	1959 1965 1985 1986 1987 2016
159	5	54	$[1.5937, 1.6113]$	IRS	1958 1962 1996 2004 2007
160	7	52	$[1.4225, 1.4227]$	IRS	1964 1977 1991 2000 2003 2012 2015
161	5	54	$[1.2362, 1.2373]$	IRS	1975 1985 1989 1998 1999
162	7	52	$[1.0838, 1.0876]$	IRS	1961 1969 1983 1984 1996 2010 2011
163	4	55	$[1.8019, 1.8041]$	IRS	1961 1993 2010 2014
164	4	55	$[1.6663, 1.6867]$	IRS	1958 1976 1993 1995
165	4	55	$[1.5071, 1.5073]$	IRS	1976 1982 2001 2011
166	4	55	$[1.9015, 1.9171]$	IRS	1961 1997 1998 2011
167	5	54	$[1.5090, 1.5356]$	IRS	1958 1968 1989 1996 2012
168	5	54	$[0.6831, 0.6855]$	DRS	1985 1986 1994 1997 2002
169	4	55	$[1.5423, 1.5510]$	IRS	1961 1966 1970 1973
170	5	54	$[1.0616, 1.0653]$	IRS	1959 1978 1982 1997 2004
171	4	55	$[1.2213, 1.2231]$	IRS	1983 1992 1993 2006
172	4	55	$[1.3845, 1.3874]$	IRS	1961 1963 1985 2005
173	5	54	$[1.0570, 1.0690]$	IRS	1963 1974 1979 1993 1994
174	4	55	$[1.6672, 1.6821]$	IRS	1960 1961 1991 2012
175	5	54	$[1.2471, 1.2559]$	IRS	1959 1964 1969 1975 2012
176	4	55	$[1.6625, 1.6768]$	IRS	1960 1976 1994 2009
177	5	54	$[0.7924, 0.7952]$	DRS	1960 1965 1972 1999 2005
178	4	55	$[1.5744, 1.5906]$	IRS	1958 1986 2001 2002
179	4	55	$[2.2574, 2.2708]$	IRS	1960 1961 1990 2014
180	4	55	$[1.1936, 1.1939]$	IRS	1960 1971 1989 2013
181	4	55	$[1.3265, 1.3444]$	IRS	1958 1966 1968 1982
182	6	53	$[0.8138, 0.8148]$	DRS	1959 1984 1985 1997 2001 2003
183	6	53	$[5.2900, 5.4888]$	IRS	1959 1961 1983 1988 1993 2003
184	5	54	$[1.0518, 1.0528]$	IRS	1960 1961 1970 1973 1994
185	5	54	$[1.0984, 1.0989]$	IRS	1958 1960 1972 1983 2000
186	4	55	$[1.0913, 1.0967]$	IRS	1962 1964 1990 1992
187	5	54	$[1.4814, 1.4921]$	IRS	1961 1962 1983 1995 2002
188	5	54	$[1.1971, 1.2006]$	IRS	1959 1966 1985 1997 2009
189	5	54	$[1.2562, 1.2612]$	IRS	1960 1961 1964 1972 2000
190	5	54	$[1.2034, 1.2042]$	IRS	1961 1962 1966 1972 1974
191	5	54	$[0.9323, 0.9328]$	DRS	1960 1980 1995 2001 2015
192	6	53	$[1.1261, 1.1358]$	IRS	1962 1963 1989 1995 1996 2008
193	6	53	$[0.9400, 0.9408]$	DRS	1970 1982 1987 1998 2005 2016
194	5	54	$[1.0557, 1.0563]$	IRS	1959 1965 1982 1996 1999

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Table C.2: Largest feasible year subset by industry, 1958–2016 (continued)

Industry	$\lvert T_i^* \rvert$	Removed	$\mathcal{K}(S_i(T_i^*))$	Regime	Kept years T_i^*
195	5	54	$[1.1640, 1.1665]$	IRS	1959 1973 1977 1989 2009
196	6	53	$[1.4311, 1.4329]$	IRS	1963 1967 1972 1977 2002 2014
197	6	53	$[0.8482, 0.8516]$	DRS	1972 1981 2000 2002 2010 2015
198	5	54	$[1.0551, 1.0597]$	IRS	1971 1972 2001 2002 2016
199	5	54	$[1.8501, 1.8574]$	IRS	1969 1978 1980 1997 2012
200	5	54	$[2.1600, 2.1725]$	IRS	1959 1983 1990 1995 2005
201	5	54	$[1.7819, 1.8004]$	IRS	1960 1967 1972 1982 1998
202	5	54	$[1.1226, 1.1233]$	IRS	1960 1968 1970 1987 1992
203	6	53	$[1.1449, 1.1454]$	IRS	1982 1984 1994 1995 1996 2003
204	5	54	$[1.5849, 1.5879]$	IRS	1975 1980 1986 2000 2001
205	5	54	$[8.9679, 12.3234]$	IRS	1958 1987 1992 2008 2015
206	4	55	$[1.1509, 1.1549]$	IRS	1969 1998 2004 2009
207	6	53	$[1.1949, 1.2063]$	IRS	1958 1970 1976 1997 2002 2015
208	5	54	$[1.2522, 1.2528]$	IRS	1958 1984 2000 2014 2016
209	5	54	$[1.5220, 1.5240]$	IRS	1958 1983 1984 1992 1997
210	5	54	$[1.6761, 1.6864]$	IRS	1976 1989 1992 1996 2009
211	6	53	$[1.1003, 1.1057]$	IRS	1958 1961 1979 1988 1998 2002
212	5	54	$[1.1625, 1.1631]$	IRS	1958 1966 1967 2000 2009
213	5	54	$[1.0368, 1.0420]$	IRS	1967 1976 1996 2006 2011
214	6	53	$[1.0738, 1.0741]$	IRS	1971 1979 1987 1996 1998 2005
215	4	55	$[0.5287, 0.5301]$	DRS	1976 1978 1998 2014
216	5	54	$[0.9037, 0.9044]$	DRS	1958 1990 1997 2007 2012
217	5	54	$[2.0994, 2.1022]$	IRS	1958 1982 1983 1987 2015
218	4	55	$[9.5110, 9.9420]$	IRS	1958 1992 2000 2004
219	4	55	$[1.7635, 1.7880]$	IRS	1958 1976 1995 1998
220	6	53	$[3.3726, 3.3760]$	IRS	1959 1962 1995 1996 1999 2003
221	5	54	$[0.9237, 0.9248]$	DRS	1960 1982 2000 2003 2010
222	5	54	$[1.2464, 1.2465]$	IRS	1960 1978 1993 1997 2009
223	5	54	$[1.1083, 1.1089]$	IRS	1960 1961 1968 1971 2016
224	5	54	$[1.2209, 1.2225]$	IRS	1960 1963 1964 1999 2009
225	5	54	$[0.9471, 0.9507]$	DRS	1959 1962 1995 2004 2016
226	5	54	$[1.7611, 1.7667]$	IRS	1969 1970 1973 1983 1994
227	5	54	$[0.7619, 0.7637]$	DRS	1964 1983 1984 2007 2010
228	6	53	$[1.1715, 1.1726]$	IRS	1959 1963 1964 1972 1982 1986
229	5	54	$[1.1687, 1.1740]$	IRS	1959 1965 1973 2008 2016
230	5	54	$[0.6839, 0.6902]$	DRS	1983 1984 1991 2006 2011
231	6	53	$[1.0885, 1.0973]$	IRS	1976 1984 1986 1997 2010 2014
232	6	53	$[0.8544, 0.8701]$	DRS	1959 1981 1985 1991 2001 2003
233	6	53	$[1.3334, 1.3425]$	IRS	1977 1983 1990 1993 2012 2013
234	7	52	$[1.5284, 1.5358]$	IRS	1960 1962 1965 1970 1984 1995 2004
235	6	53	$[1.6832, 1.6949]$	IRS	1962 1965 1990 1991 2012 2015
236	4	55	$[1.1378, 1.1443]$	IRS	1958 1983 1985 2008
237	5	54	$[1.3482, 1.3488]$	IRS	1988 1998 2004 2015 2016
238	5	54	$[1.6387, 1.6454]$	IRS	1958 1964 1979 1989 2005
239	6	53	$[1.9543, 1.9769]$	IRS	1966 1981 1988 1994 2001 2002
240	5	54	$[1.4447, 1.4486]$	IRS	1960 1975 1987 1990 2000
241	5	54	$[0.7793, 0.7878]$	DRS	1979 1994 1998 2004 2010
242	5	54	$[1.3262, 1.3282]$	IRS	1974 1982 1983 1989 2003

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Table C.2: Largest feasible year subset by industry, 1958–2016 (continued)

Industry	$\lvert T_i^* \rvert$	Removed	$\mathcal{K}(S_i(T_i^*))$	Regime	Kept years T_i^*
243	6	53	$[2.1901, 2.2070]$	IRS	1974 1982 1984 1987 2014 2016
244	6	53	$[1.0407, 1.0418]$	IRS	1959 1966 1971 1981 1989 2001
245	8	51	$[1.1175, 1.1195]$	IRS	1958 1965 1967 1971 1982 1985 1993 2016
246	6	53	$[0.8359, 0.8405]$	DRS	1958 1969 1982 1993 2001 2002
247	6	53	$[1.0817, 1.0849]$	IRS	1960 1965 1978 1983 2000 2010
248	6	53	$[0.9267, 0.9283]$	DRS	1975 1982 1985 1994 2012 2016
249	6	53	$[1.1876, 1.1953]$	IRS	1958 1983 1987 1990 1997 2011
250	7	52	$[1.0086, 1.0119]$	IRS	1958 1967 1979 1990 1998 2003 2014
251	5	54	$[2.3303, 2.3566]$	IRS	1979 1984 1986 2006 2016
252	6	53	$[1.2783, 1.2785]$	IRS	1961 1963 1974 1987 1992 2010
253	5	54	$[1.0977, 1.1073]$	IRS	1961 1969 1992 1997 2013
254	6	53	$[1.1043, 1.1050]$	IRS	1958 1971 1984 1995 2006 2014
255	6	53	$[1.2047, 1.2060]$	IRS	1975 1977 1990 1996 2002 2006
256	5	54	$[0.7695, 0.7758]$	DRS	1974 1982 1995 1997 2004
257	5	54	$[1.0436, 1.0441]$	IRS	1960 1975 1979 2003 2016
258	5	54	$[0.7792, 0.7799]$	DRS	1958 1976 1994 1997 2015
259	5	54	$[0.9458, 0.9493]$	DRS	1973 1977 1980 1986 2016
260	7	52	$[1.0835, 1.0853]$	IRS	1963 1965 1971 1981 1985 2006 2010
261	8	51	$[1.3221, 1.3236]$	IRS	1962 1963 1967 1972 1986 1992 2006 2013
262	5	54	$[1.0331, 1.0350]$	IRS	1958 1982 1994 1997 2009
263	7	52	$[2.1290, 2.1400]$	IRS	1973 1983 1986 1988 1991 1998 2001
264	9	50	$[1.7717, 1.7731]$	IRS	1960 1975 1977 1982 1983 1985 1993 1994 1997
265	6	53	$[1.7509, 1.7572]$	IRS	1968 1975 1976 1983 1985 1998
266	7	52	$[1.4930, 1.4957]$	IRS	1961 1965 1968 1989 1995 2003 2008
267	6	53	$[1.1012, 1.1085]$	IRS	1977 1979 1984 1989 1995 2013
268	6	53	$[1.0824, 1.0835]$	IRS	1977 1978 1985 1993 1995 2016
269	5	54	$[3.4971, 3.5203]$	IRS	1964 1975 1980 1984 2006
270	9	50	$[1.1651, 1.1750]$	IRS	1959 1961 1974 1980 1985 1987 1998 2005 2010
271	5	54	$[3.1045, 3.1344]$	IRS	1974 1981 1993 1998 2000
272	7	52	$[1.4146, 1.4264]$	IRS	1961 1962 1979 1983 1991 1992 2011
273	6	53	$[1.3851, 1.3997]$	IRS	1961 1966 1969 2000 2014 2015
274	8	51	$[1.0108, 1.0144]$	IRS	1959 1961 1962 1975 1978 1993 1999 2011
275	6	53	$[0.9982, 0.9996]$	DRS	1972 1985 1989 1995 2000 2005
276	6	53	$[1.1532, 1.1639]$	IRS	1979 1986 1996 1998 2001 2002
277	8	51	$[1.5590, 1.5812]$	IRS	1963 1968 1984 1989 1990 1991 1998 2015
278	7	52	$[1.1573, 1.1726]$	IRS	1960 1965 1995 1996 1998 2005 2012
279	6	53	$[0.7473, 0.7561]$	DRS	1978 1984 1986 1999 2000 2008
280	5	54	$[1.0571, 1.0626]$	IRS	1959 1964 1974 1987 1995
281	7	52	$[1.2376, 1.2392]$	IRS	1958 1966 1970 1983 1991 2009 2010
282	6	53	$[1.2074, 1.2107]$	IRS	1975 1980 1992 1998 2006 2014
283	6	53	$[0.8706, 0.8730]$	DRS	1975 1987 1996 1997 2015 2016
284	7	52	$[1.3656, 1.3716]$	IRS	1959 1963 1965 1973 1996 2005 2016
285	6	53	$[1.6598, 1.6639]$	IRS	1961 1975 1977 1981 1988 1992
286	6	53	$[1.4444, 1.4602]$	IRS	1961 1965 1981 1995 2003 2004
287	6	53	$[0.7939, 0.7965]$	DRS	1967 1979 1997 1999 2001 2002
288	7	52	$[1.1475, 1.1478]$	IRS	1958 1967 1970 1987 1994 2000 2001
289	5	54	$[1.2813, 1.2868]$	IRS	1958 1963 1969 1985 1989
290	5	54	$[1.3778, 1.3784]$	IRS	1958 1964 1967 1994 2005

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Table C.2: Largest feasible year subset by industry, 1958–2016 (continued)

Industry	$\lvert T_i^* \rvert$	Removed	$\mathcal{K}(S_i(T_i^*))$	Regime	Kept years T_i^*
291	5	54	$[3.0230, 3.0305]$	IRS	1963 1964 1984 1994 1995
292	5	54	$[1.8027, 1.8171]$	IRS	1960 1964 1971 1996 2000
293	5	54	$[65.6230, 76.6604]$	IRS	1959 1963 1978 1993 2002
294	4	55	$[0.6588, 0.6704]$	DRS	1994 1998 2009 2010
295	4	55	$[2.3530, 2.3568]$	IRS	1963 1964 1974 2014
296	6	53	$[2.1078, 2.1210]$	IRS	1958 1978 1979 1989 1994 2000
297	5	54	$[1.3638, 1.3653]$	IRS	1964 1966 1987 2015 2016
298	5	54	$[1.4672, 1.4699]$	IRS	1961 1973 1981 1992 1993
299	6	53	$[1.1646, 1.1659]$	IRS	1962 1969 1973 1978 1987 1993
300	6	53	$[2.0043, 2.0188]$	IRS	1959 1963 1976 1997 2003 2012
301	6	53	$[1.3908, 1.3943]$	IRS	1964 1975 1983 1986 2002 2004
302	4	55	$[1.4105, 1.4121]$	IRS	1958 1982 1993 1999
303	5	54	$[1.3136, 1.3145]$	IRS	1958 1988 1997 2000 2009
304	5	54	$[1.4216, 1.4315]$	IRS	1968 1970 1982 1996 1998
305	5	54	$[0.7349, 0.7467]$	DRS	1958 1960 1983 1999 2001
306	6	53	$[1.3433, 1.3527]$	IRS	1959 1968 1971 1972 1989 2001
307	9	50	$[1.2755, 1.2766]$	IRS	1971 1977 1982 1990 1995 2005 2006 2010 2016
308	6	53	$[1.2485, 1.2526]$	IRS	1962 1972 1994 1998 2000 2014
309	6	53	$[1.2527, 1.2533]$	IRS	1961 1966 1972 1973 1998 2003
310	6	53	$[1.3608, 1.3620]$	IRS	1958 1962 1972 1996 1997 1998
311	5	54	$[1.0328, 1.0419]$	IRS	1958 1962 1997 2001 2013
312	5	54	$[1.0879, 1.0883]$	IRS	1962 1965 1971 1996 2014
313	8	51	$[1.0747, 1.0748]$	IRS	1958 1960 1966 1979 1990 1996 1999 2002
314	5	54	$[1.2256, 1.2276]$	IRS	1959 1961 2003 2009 2015
315	6	53	$[1.2680, 1.2691]$	IRS	1958 1962 1970 1982 2012 2014
316	7	52	$[1.0888, 1.0922]$	IRS	1960 1965 1967 1982 1986 1998 1999
317	4	55	$[1.1764, 1.1814]$	IRS	1982 1984 1988 1996
318	4	55	$[1.0517, 1.0573]$	IRS	1983 1997 2000 2003
319	5	54	$[1.2539, 1.2557]$	IRS	1970 1971 1982 1985 2007
320	5	54	$[1.1595, 1.1623]$	IRS	1964 1966 1969 1995 2013
321	5	54	$[1.9454, 1.9465]$	IRS	1966 1985 1995 1996 2005
322	5	54	$[1.1348, 1.1359]$	IRS	1969 1970 1981 1990 2000
323	6	53	$[1.0405, 1.0680]$	IRS	1963 1982 1992 2000 2012 2015
324	6	53	$[1.3498, 1.3561]$	IRS	1961 1967 1969 1996 2000 2001
325	5	54	$[1.6488, 1.6729]$	IRS	1963 1982 1993 2013 2014
326	6	53	$[1.2592, 1.2634]$	IRS	1971 1979 1984 1992 1999 2015
327	4	55	$[0.9826, 0.9852]$	DRS	1992 1995 1996 2007
328	5	54	$[1.0209, 1.0319]$	IRS	1981 1990 1991 2015 2016
329	5	54	$[1.0410, 1.0454]$	IRS	1976 1982 1989 2006 2009
330	5	54	$[1.2596, 1.2619]$	IRS	1964 1970 1983 1996 2008
331	5	54	$[1.0640, 1.0660]$	IRS	1960 1961 2005 2013 2016
332	5	54	$[1.3626, 1.3763]$	IRS	1964 1969 1970 1996 2002
333	5	54	$[0.9107, 0.9132]$	DRS	1975 1976 1989 2007 2008
334	6	53	$[1.0518, 1.0544]$	IRS	1958 1961 1973 1981 1998 2005
335	5	54	$[1.1555, 1.1597]$	IRS	1980 1984 1994 2004 2010
336	6	53	$[1.0512, 1.0533]$	IRS	1981 1982 1984 1988 2005 2009
337	6	53	$[1.3673, 1.3681]$	IRS	1960 1961 1973 1983 2004 2007
338	5	54	$[0.9356, 0.9418]$	DRS	1984 1985 1987 1994 2003

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Table C.2: Largest feasible year subset by industry, 1958–2016 (continued)

Industry	$\lvert T_i^* \rvert$	Removed	$\mathcal{K}(S_i(T_i^*))$	Regime	Kept years T_i^*
339	6	53	$[1.2337, 1.2399]$	IRS	1960 1962 1963 1964 1971 1992
340	6	53	$[1.2033, 1.2074]$	IRS	1958 1964 1969 1988 2002 2009
341	7	52	$[1.0714, 1.0727]$	IRS	1960 1968 1975 1998 1999 2000 2015
342	5	54	$[1.0729, 1.0748]$	IRS	1960 1997 2000 2006 2011
343	8	51	$[1.1631, 1.1647]$	IRS	1960 1966 1973 1976 1986 1999 2002 2010
344	5	54	$[1.0988, 1.1005]$	IRS	1972 1975 1979 2005 2010
345	5	54	$[2.3905, 2.4093]$	IRS	1959 1961 1965 1998 2009
346	4	55	$[1.1342, 1.1356]$	IRS	1980 1987 2006 2010
347	7	52	$[1.2455, 1.2465]$	IRS	1974 1977 1979 1990 1992 1994 2004
348	6	53	$[1.1739, 1.1773]$	IRS	1976 1977 1990 2007 2015 2016
349	5	54	$[0.6869, 0.6874]$	DRS	1987 1990 1995 2000 2003
350	7	52	$[1.4266, 1.4437]$	IRS	1983 1990 1992 1994 1998 2000 2014
351	5	54	$[0.8085, 0.8143]$	DRS	1962 1995 2003 2012 2016
352	4	55	$[1.5715, 1.5767]$	IRS	1958 1981 1988 2007
353	6	53	$[0.8687, 0.8688]$	DRS	1979 1980 1990 2000 2008 2012
354	5	54	$[0.8036, 0.8044]$	DRS	1995 1996 1999 2003 2007
355	5	54	$[1.1016, 1.1032]$	IRS	1964 1965 1968 2012 2016
356	5	54	$[0.8632, 0.8661]$	DRS	1959 1979 2000 2007 2010
357	6	53	$[0.7915, 0.7934]$	DRS	1969 1982 1983 1997 2000 2014
358	5	54	$[0.7570, 0.7659]$	DRS	1967 1979 1988 1993 2011
359	5	54	$[2.9543, 3.0683]$	IRS	1958 1962 1974 1993 2007
360	4	55	$[1.5685, 1.6029]$	IRS	1982 1988 2010 2013
361	6	53	$[1.4398, 1.4398]$	IRS	1960 1968 1973 1974 1992 2008

Notes: Industry is the industry identifier used throughout the paper; $\lvert T_i^* \rvert$ is the size of the largest subset of years for which $\mathcal{K}(S_i(T)) \neq \varnothing$ (Section 4.4); Removed is $\lvert T_i \rvert - \lvert T_i^* \rvert$, the number of years dropped to achieve this; $\mathcal{K}(S_i(T_i^*)) = [\underline{k}, \overline{k}]$ is the identified set over the retained years; Regime is IRS/DRS required if the interval lies entirely above/below one, or CRS admissible if $1 \in \mathcal{K}(S_i(T_i^*))$; Kept years T_i^* lists the retained years. If multiple year subsets attain the same maximum cardinality, one such subset is reported. Capital is priced using the user cost of capital at $r = 8\%$.

Table C.3: Largest feasible year subset by industry, 1958–2016

Industry	$\lvert T_i^* \rvert$	Removed	$\mathcal{K}(S_i(T_i^*))$	Regime	Kept years T_i^*
1	3	56	$[5.7203, 5.7585]$	IRS	1967 2001 2002
2	4	55	$[1.2572, 1.2606]$	IRS	1959 1970 1988 1989
3	3	56	$[1.6584, 1.6611]$	IRS	1977 1980 2015
4	6	53	$[3.5976, 3.6123]$	IRS	1968 1980 1982 1991 2008 2009
5	4	55	$[0.1810, 0.1822]$	DRS	1981 1982 1984 2005
6	5	54	$[1.6059, 1.6314]$	IRS	1958 1959 1961 1984 2016
7	4	55	$[1.1341, 1.1442]$	IRS	1959 1969 1983 1986
8	5	54	$[1.2152, 1.2187]$	IRS	1958 1966 1980 1990 1993
9	5	54	$[1.3366, 1.3382]$	IRS	1961 1962 1966 2001 2002
10	5	54	$[1.1994, 1.2004]$	IRS	1960 1962 1970 1986 1995
11	5	54	$[1.2814, 1.2817]$	IRS	1958 1961 1977 1992 1999
12	7	52	$[1.9934, 2.0011]$	IRS	1958 1964 1967 1978 1988 2005 2009
13	4	55	$[1.4541, 1.4783]$	IRS	1965 1974 1993 2006
14	6	53	$[1.7763, 1.7960]$	IRS	1960 1964 1973 1985 1988 2014
15	4	55	$[2.0017, 2.0125]$	IRS	1964 1970 2005 2009
16	4	55	$[1.3370, 1.3407]$	IRS	1963 1979 1999 2007
17	6	53	$[2.1147, 2.1623]$	IRS	1960 1963 1976 1995 1996 2008
18	5	54	$[1.9306, 1.9730]$	IRS	1958 1959 1984 1991 1994
19	4	55	$[1.3713, 1.3725]$	IRS	1980 2000 2006 2016
20	5	54	$[24.1843, 61.7477]$	IRS	1958 1969 1977 1987 1995
21	4	55	$[0.9240, 0.9250]$	DRS	1963 1964 1972 1988
22	6	53	$[0.9555, 0.9556]$	DRS	1970 1971 1977 1980 1998 1999
23	5	54	$[0.9271, 0.9405]$	DRS	1958 1961 1967 1988 2009
24	4	55	$[3.5289, 3.5838]$	IRS	1959 1986 2005 2006
25	4	55	$[1.7038, 1.7112]$	IRS	1958 1969 1971 1993
26	4	55	$[1.1765, 1.1865]$	IRS	1959 1960 1980 1994
27	4	55	$[3.2343, 3.3440]$	IRS	1959 1962 1992 2008
28	4	55	$[1.3396, 1.3478]$	IRS	1976 1985 2008 2009
29	5	54	$[0.3099, 0.3115]$	DRS	1964 1967 1974 1998 2010
30	5	54	$[1.0301, 1.0413]$	IRS	1978 1981 1985 2011 2014
31	4	55	$[1.1267, 1.1359]$	IRS	1958 1966 2001 2005
32	5	54	$[1.4509, 1.4618]$	IRS	1959 1966 1967 1973 2010
33	6	53	$[1.2267, 1.2312]$	IRS	1958 1961 1973 1983 1997 2016
34	5	54	$[1.4701, 1.4733]$	IRS	1958 1969 1973 1993 2016
35	4	55	$[1.5361, 1.5612]$	IRS	1958 1973 1979 1988
36	5	54	$[1.6305, 1.6348]$	IRS	1958 1988 1995 2003 2010
37	3	56	$[2.6449, 2.6745]$	IRS	1969 1972 2016
38	4	55	$[1.9550, 1.9572]$	IRS	1964 1968 1970 1986
39	4	55	$[1.5175, 1.5420]$	IRS	1971 1974 1982 2002
40	4	55	$[1.2571, 1.2811]$	IRS	1958 1977 2012 2014
41	5	54	$[1.1169, 1.1186]$	IRS	1966 1967 1970 2003 2004
42	4	55	$[1.2938, 1.3064]$	IRS	1963 1970 1992 2012
43	6	53	$[0.8180, 0.8429]$	DRS	1958 1967 1972 1986 2000 2010
44	6	53	$[1.1462, 1.1520]$	IRS	1962 1963 1966 1967 1990 1993
45	5	54	$[1.8329, 2.0210]$	IRS	1974 1979 1980 1984 2009
46	5	54	$[5.1796, 5.4476]$	IRS	1960 1961 1963 2005 2013
47	4	55	$[1.2732, 1.2763]$	IRS	1959 1962 2009 2013
48	4	55	$[7.4634, 8.1268]$	IRS	1960 1974 1985 2015

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Table C.3: Largest feasible year subset by industry, 1958–2016 (continued)

Industry	$\lvert T_i^* \rvert$	Removed	$\mathcal{K}(S_i(T_i^*))$	Regime	Kept years T_i^*
49	5	54	$[1.3001, 1.3035]$	IRS	1979 1982 1984 1991 2014
50	7	52	$[1.1695, 1.1794]$	IRS	1963 1967 1971 1979 1989 1993 2001
51	5	54	$[0.6012, 0.6029]$	DRS	1971 1977 1985 1986 2002
52	5	54	$[0.5757, 0.5814]$	DRS	1959 1961 1976 1978 2014
53	5	54	$[1.3219, 1.3267]$	IRS	1961 1967 1978 1993 2002
54	4	55	$[1.2802, 1.2817]$	IRS	1958 1967 1969 1978
55	7	52	$[0.9149, 0.9162]$	DRS	1959 1962 1964 1980 1983 1988 2009
56	5	54	$[2.2047, 2.2088]$	IRS	1960 1982 1986 1989 2010
57	6	53	$[1.5932, 1.6045]$	IRS	1962 1966 1973 1981 1995 2006
58	5	54	$[1.4918, 1.4971]$	IRS	1974 1980 1988 1991 2002
59	6	53	$[1.4676, 1.4755]$	IRS	1958 1960 1971 2010 2013 2016
60	5	54	$[1.8804, 1.8868]$	IRS	1960 1961 1989 2001 2003
61	6	53	$[1.6056, 1.6123]$	IRS	1958 1962 1972 1974 1995 1999
62	6	53	$[0.8327, 0.8329]$	DRS	1983 1984 1993 2003 2010 2011
63	5	54	$[0.8927, 0.8934]$	DRS	1987 1999 2001 2004 2008
64	4	55	$[0.4482, 0.4577]$	DRS	1977 1981 2010 2014
65	5	54	$[0.6303, 0.6370]$	DRS	1970 1972 1989 1992 1999
66	5	54	$[4.3823, 4.3886]$	IRS	1963 1974 1979 1993 2001
67	4	55	$[0.5758, 0.5836]$	DRS	2000 2008 2013 2016
68	6	53	$[2.5800, 2.6000]$	IRS	1960 1965 1967 1968 1978 1998
69	4	55	$[1.2599, 1.2646]$	IRS	1958 1981 2003 2006
70	4	55	$[0.8426, 0.8442]$	DRS	1967 1979 2001 2007
71	4	55	$[0.7316, 0.7342]$	DRS	1976 1988 2004 2012
72	4	55	$[1.0063, 1.0085]$	IRS	1995 1997 2004 2016
73	5	54	$[1.3045, 1.3086]$	IRS	1965 1972 1982 1987 2003
74	4	55	$[0.9529, 0.9542]$	DRS	1976 1985 2006 2014
75	4	55	$[2.4354, 2.4383]$	IRS	1961 1963 1969 2005
76	5	54	$[1.6950, 1.7023]$	IRS	1960 1962 1965 1991 2003
77	4	55	$[1.1753, 1.1772]$	IRS	1958 1966 1993 2014
78	5	54	$[1.2153, 1.2156]$	IRS	1960 1963 1974 1992 2016
79	5	54	$[1.8077, 1.8296]$	IRS	1959 1965 1974 1997 2008
80	5	54	$[0.8724, 0.8759]$	DRS	1959 1964 1982 1997 2012
81	5	54	$[1.6906, 1.6935]$	IRS	1961 1972 1981 1986 2014
82	5	54	$[0.9944, 0.9960]$	DRS	1958 1959 1965 1999 2009
83	5	54	$[0.9732, 0.9740]$	DRS	1982 1996 1997 2003 2016
84	5	54	$[1.2048, 1.2072]$	IRS	1959 1960 1999 2001 2004
85	4	55	$[0.8119, 0.8123]$	DRS	1965 1966 1969 2003
86	5	54	$[2.3688, 2.4541]$	IRS	1977 1989 2000 2004 2016
87	4	55	$[1.9247, 1.9380]$	IRS	1958 1963 1993 2008
88	5	54	$[1.6287, 1.6365]$	IRS	1958 1975 1981 1985 2000
89	4	55	$[1.8397, 1.8511]$	IRS	1958 1960 1977 1993
90	4	55	$[2.0912, 2.0982]$	IRS	1958 1960 1996 1997
91	5	54	$[1.0514, 1.0547]$	IRS	1965 1968 1973 1999 2003
92	5	54	$[1.4160, 1.4229]$	IRS	1961 1965 1969 1990 1997
93	6	53	$[1.1376, 1.1445]$	IRS	1958 1960 1961 1966 1983 1989
94	4	55	$[1.2439, 1.2451]$	IRS	1958 1963 1964 1997
95	5	54	$[1.2942, 1.2979]$	IRS	1958 1970 1990 1999 2009
96	5	54	$[1.0057, 1.0067]$	IRS	1960 1966 1967 1986 1992

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Table C.3: Largest feasible year subset by industry, 1958–2016 (continued)

Industry	$\lvert T_i^* \rvert$	Removed	$\mathcal{K}(S_i(T_i^*))$	Regime	Kept years T_i^*
97	5	54	$[1.4163, 1.4191]$	IRS	1967 1970 1973 1999 2002
98	4	55	$[0.5463, 0.5463]$	DRS	1992 2001 2013 2016
99	4	55	$[1.7100, 1.7399]$	IRS	1974 1977 1979 2005
100	5	54	$[1.2772, 1.2889]$	IRS	1980 1982 1985 1988 1997
101	4	55	$[1.2877, 1.2935]$	IRS	1979 1981 1984 1988
102	4	55	$[0.9713, 0.9747]$	DRS	1975 1976 1978 2009
103	5	54	$[0.9174, 0.9212]$	DRS	1978 1979 1981 1982 1993
104	4	55	$[2.2542, 2.2632]$	IRS	1958 1964 1966 1968
105	4	55	$[0.2811, 0.2824]$	DRS	1982 1983 1986 2015
106	5	54	$[1.8539, 1.9568]$	IRS	1958 1964 1968 1972 1998
107	4	55	$[1.4786, 1.4813]$	IRS	1958 1959 1969 2016
108	5	54	$[1.9832, 2.0404]$	IRS	1958 1959 1989 2003 2015
109	6	53	$[0.9607, 0.9610]$	DRS	1964 1968 1969 1990 2000 2001
110	5	54	$[1.8769, 1.8831]$	IRS	1961 1964 1973 1975 1992
111	5	54	$[1.5510, 1.5545]$	IRS	1958 1960 1967 1969 1973
112	5	54	$[1.5421, 1.5476]$	IRS	1964 1968 1969 1997 2016
113	4	55	$[1.5747, 1.5767]$	IRS	1959 1967 1969 1973
114	4	55	$[1.6318, 1.6490]$	IRS	1958 1964 1969 1973
115	4	55	$[1.6044, 1.6137]$	IRS	1958 1967 1996 2009
116	4	55	$[2.0159, 2.1251]$	IRS	1958 1961 1990 1998
117	4	55	$[2.9190, 2.9484]$	IRS	1958 1988 1999 2001
118	4	55	$[2.0174, 2.0200]$	IRS	1961 1980 1981 2008
119	3	56	$[4.9585, 5.0275]$	IRS	1958 1991 2009
120	4	55	$[1.6263, 1.6393]$	IRS	1959 1963 1988 2009
121	4	55	$[1.7017, 1.7088]$	IRS	1961 1978 1995 2002
122	5	54	$[0.9789, 0.9825]$	DRS	1972 1990 1994 2004 2007
123	5	54	$[1.2868, 1.2886]$	IRS	1966 1968 1991 1997 2013
124	5	54	$[1.3231, 1.3250]$	IRS	1959 1972 1974 1992 2011
125	5	54	$[1.6693, 1.6735]$	IRS	1959 1977 1994 2004 2009
126	4	55	$[1.1896, 1.1904]$	IRS	1958 1989 1994 2010
127	4	55	$[1.8940, 1.8944]$	IRS	1960 1961 1966 2000
128	5	54	$[2.6373, 2.7194]$	IRS	1958 1964 1986 1991 2015
129	5	54	$[1.0662, 1.0688]$	IRS	1975 1977 1978 1988 2000
130	6	53	$[0.7570, 0.7619]$	DRS	1980 1987 1990 1998 2000 2006
131	4	55	$[0.9228, 0.9241]$	DRS	1958 1960 2004 2014
132	4	55	$[1.0804, 1.0970]$	IRS	1960 1965 2002 2003
133	6	53	$[1.1927, 1.1939]$	IRS	1958 1959 1977 1978 1992 2007
134	7	52	$[1.5038, 1.5339]$	IRS	1958 1961 1962 1965 1986 1996 1999
135	5	54	$[1.1445, 1.1463]$	IRS	1960 1985 1995 2006 2015
136	4	55	$[1.2853, 1.2856]$	IRS	1960 1970 1994 2000
137	5	54	$[1.4341, 1.4391]$	IRS	1959 1962 1984 1985 1987
138	5	54	$[1.2208, 1.2225]$	IRS	1958 1962 1978 1991 2003
139	5	54	$[1.3657, 1.3761]$	IRS	1958 1961 1971 1999 2001
140	5	54	$[1.1925, 1.1945]$	IRS	1958 1961 1962 1978 2000
141	5	54	$[1.2541, 1.2550]$	IRS	1958 1959 1979 1994 2002
142	6	53	$[1.2721, 1.2736]$	IRS	1958 1980 1985 1991 1993 2000
143	7	52	$[1.2736, 1.2743]$	IRS	1960 1980 1985 1991 1993 2001 2003
144	6	53	$[1.2372, 1.2376]$	IRS	1958 1962 1965 1977 1988 1996

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Table C.3: Largest feasible year subset by industry, 1958–2016 (continued)

Industry	$\lvert T_i^* \rvert$	Removed	$\mathcal{K}(S_i(T_i^*))$	Regime	Kept years T_i^*
145	5	54	$[1.2737, 1.2761]$	IRS	1958 1963 1965 2000 2006
146	6	53	$[1.2994, 1.2997]$	IRS	1958 1959 1962 1989 1994 2001
147	4	55	$[5.9386, 6.2271]$	IRS	1958 1961 1996 2006
148	6	53	$[1.0797, 1.0886]$	IRS	1958 1960 1969 1990 1993 2007
149	5	54	$[1.7015, 1.7027]$	IRS	1958 1962 1967 1986 2002
150	5	54	$[1.5073, 1.5307]$	IRS	1958 1962 1969 1999 2000
151	4	55	$[0.8172, 0.8187]$	DRS	1992 2001 2003 2010
152	3	56	$[0.4318, 0.4345]$	DRS	1973 1981 2009
153	4	55	$[2.0332, 2.0580]$	IRS	1958 1968 1974 1979
154	5	54	$[1.5233, 1.5268]$	IRS	1958 1959 1964 1997 2005
155	4	55	$[0.5263, 0.5267]$	DRS	1968 1997 2013 2016
156	5	54	$[1.2858, 1.2944]$	IRS	1978 1995 1996 1999 2002
157	7	52	$[1.3778, 1.3816]$	IRS	1958 1974 1977 1990 1995 2011 2014
158	5	54	$[1.4187, 1.4192]$	IRS	1958 1960 1971 1985 2015
159	5	54	$[1.6008, 1.6201]$	IRS	1958 1959 1966 2000 2006
160	5	54	$[1.4200, 1.4231]$	IRS	1961 1981 1990 2000 2005
161	5	54	$[1.3529, 1.3550]$	IRS	1962 1980 1999 2004 2006
162	6	53	$[0.9563, 0.9593]$	DRS	1967 1985 1992 1997 2010 2011
163	5	54	$[0.7800, 0.7811]$	DRS	1989 1993 2001 2002 2011
164	4	55	$[1.4807, 1.4832]$	IRS	1958 1988 1991 1992
165	5	54	$[1.3136, 1.3185]$	IRS	1976 1977 1982 1991 2016
166	4	55	$[1.1956, 1.2104]$	IRS	1975 1976 1985 2006
167	4	55	$[1.5587, 1.5686]$	IRS	1958 1968 1996 2015
168	4	55	$[0.8878, 0.8910]$	DRS	1979 1984 2002 2003
169	4	55	$[0.9992, 1.0079]$	CRS	1968 1989 1991 2008
170	5	54	$[0.9232, 0.9234]$	DRS	1984 1990 1994 1999 2002
171	4	55	$[1.1460, 1.1462]$	IRS	1963 1966 1990 1995
172	5	54	$[1.3299, 1.3331]$	IRS	1958 1959 1970 1997 2002
173	5	54	$[1.1621, 1.1656]$	IRS	1958 1974 1992 1994 1995
174	5	54	$[1.6571, 1.6641]$	IRS	1960 1961 1994 1998 2010
175	6	53	$[1.2273, 1.2301]$	IRS	1960 1964 1969 1975 1993 2016
176	4	55	$[1.6867, 1.6906]$	IRS	1960 1976 2009 2016
177	4	55	$[0.9323, 0.9345]$	DRS	1958 1959 1998 2007
178	3	56	$[2.5266, 2.6113]$	IRS	1958 1982 2003
179	4	55	$[1.1773, 1.1783]$	IRS	1958 1961 1989 2009
180	4	55	$[1.2266, 1.2332]$	IRS	1960 1975 1988 2014
181	4	55	$[1.3074, 1.3367]$	IRS	1958 1964 1970 1973
182	5	54	$[0.8769, 0.8816]$	DRS	1961 1983 1987 2001 2002
183	6	53	$[0.9521, 0.9525]$	DRS	1959 1981 1990 1996 1999 2003
184	5	54	$[1.1168, 1.1189]$	IRS	1961 1967 1971 2001 2002
185	5	54	$[1.1203, 1.1227]$	IRS	1962 1966 1969 1971 2001
186	4	55	$[4.5174, 4.5714]$	IRS	1987 1993 1995 2016
187	4	55	$[1.1787, 1.1818]$	IRS	1958 1959 1995 2002
188	4	55	$[1.2476, 1.2550]$	IRS	1962 1974 2005 2016
189	5	54	$[1.2282, 1.2333]$	IRS	1960 1961 1964 1970 2009
190	4	55	$[1.1374, 1.1396]$	IRS	1960 1982 1998 1999
191	5	54	$[1.1803, 1.1822]$	IRS	1974 1980 1982 1998 2009
192	5	54	$[1.3175, 1.3299]$	IRS	1960 1970 1973 1996 2004

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Table C.3: Largest feasible year subset by industry, 1958–2016 (continued)

Industry	$\lvert T_i^* \rvert$	Removed	$\mathcal{K}(S_i(T_i^*))$	Regime	Kept years T_i^*
193	5	54	$[0.9219, 0.9227]$	DRS	1982 1987 1998 2002 2009
194	4	55	$[1.0615, 1.0639]$	IRS	1959 1967 1982 2000
195	4	55	$[1.2814, 1.2936]$	IRS	1959 1987 1996 2003
196	4	55	$[1.6174, 1.6318]$	IRS	1959 1966 2013 2016
197	5	54	$[0.8225, 0.8245]$	DRS	1972 1980 1982 1997 2001
198	4	55	$[1.3565, 1.3607]$	IRS	1960 1961 1997 2002
199	4	55	$[1.6760, 1.6954]$	IRS	1969 1981 1998 2003
200	4	55	$[3.4559, 3.4663]$	IRS	1959 1960 1993 2005
201	4	55	$[3.0719, 3.0989]$	IRS	1959 1960 1992 2001
202	4	55	$[1.0696, 1.0749]$	IRS	1958 1960 1978 1991
203	4	55	$[1.1853, 1.1957]$	IRS	1961 1963 1968 1999
204	4	55	$[2.8962, 2.9093]$	IRS	1959 1962 1981 2002
205	4	55	$[1.3671, 1.3705]$	IRS	1982 1984 1994 2001
206	4	55	$[1.2030, 1.2252]$	IRS	1983 1991 1998 2006
207	5	54	$[1.2244, 1.2268]$	IRS	1960 1966 1967 2003 2015
208	5	54	$[1.3114, 1.3130]$	IRS	1961 1982 1993 2006 2016
209	5	54	$[1.9033, 1.9175]$	IRS	1958 1982 1992 2005 2008
210	6	53	$[1.9447, 1.9463]$	IRS	1967 1968 1980 2004 2005 2015
211	5	54	$[1.1048, 1.1097]$	IRS	1958 1961 1979 1988 1997
212	5	54	$[1.1907, 1.2072]$	IRS	1958 1965 1998 2002 2003
213	5	54	$[0.9981, 0.9996]$	DRS	1973 1980 1992 1993 2003
214	5	54	$[1.0396, 1.0429]$	IRS	1965 1990 1996 1998 2006
215	4	55	$[0.8100, 0.8123]$	DRS	1968 1992 2000 2004
216	4	55	$[0.7276, 0.7297]$	DRS	1958 1960 2001 2009
217	5	54	$[0.8523, 0.8551]$	DRS	1982 1988 1994 1996 1998
218	4	55	$[16.0844, 17.9179]$	IRS	1963 1977 1992 2004
219	4	55	$[1.7232, 1.7696]$	IRS	1958 1976 1995 1998
220	5	54	$[1.6319, 1.6401]$	IRS	1960 1973 1974 2010 2016
221	4	55	$[1.0846, 1.0848]$	IRS	1985 1987 1988 2009
222	5	54	$[1.2098, 1.2111]$	IRS	1960 1988 1992 1999 2000
223	4	55	$[1.0565, 1.0662]$	IRS	1960 1961 1966 2009
224	4	55	$[1.1838, 1.1848]$	IRS	1960 1963 1964 2003
225	5	54	$[1.0619, 1.0643]$	IRS	1961 1970 1980 1993 1997
226	5	54	$[0.7832, 0.7850]$	DRS	1983 1988 1989 1996 2014
227	5	54	$[0.8066, 0.8077]$	DRS	1960 1989 1991 2006 2009
228	5	54	$[1.1107, 1.1113]$	IRS	1959 1960 1969 1992 2016
229	5	54	$[2.1470, 2.1578]$	IRS	1973 1974 1980 1986 2014
230	5	54	$[1.3316, 1.3390]$	IRS	1959 1962 1983 2007 2016
231	6	53	$[1.1557, 1.1633]$	IRS	1958 1973 1984 1993 1997 2008
232	6	53	$[1.0618, 1.0651]$	IRS	1958 1982 1983 1985 2003 2004
233	8	51	$[1.6506, 1.6519]$	IRS	1966 1983 1988 1992 1996 2014 2015 2016
234	6	53	$[1.4908, 1.5044]$	IRS	1958 1961 1965 1985 1996 1999
235	6	53	$[1.8866, 1.8919]$	IRS	1963 1970 1973 1986 2011 2013
236	4	55	$[1.0871, 1.0891]$	IRS	1958 1982 1986 2004
237	4	55	$[0.6306, 0.6309]$	DRS	1978 1982 1988 2016
238	5	54	$[2.0316, 2.0362]$	IRS	1966 1976 1990 1993 2009
239	5	54	$[1.6071, 1.6297]$	IRS	1964 1968 1976 1992 2000
240	6	53	$[2.0331, 2.0412]$	IRS	1979 1980 1990 1994 1997 2001

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Table C.3: Largest feasible year subset by industry, 1958–2016 (continued)

Industry	$\lvert T_i^* \rvert$	Removed	$\mathcal{K}(S_i(T_i^*))$	Regime	Kept years T_i^*
241	5	54	$[2.2122, 2.2185]$	IRS	1959 1983 1993 2005 2007
242	5	54	$[1.2379, 1.2409]$	IRS	1966 1973 1982 1987 2002
243	5	54	$[1.4248, 1.4252]$	IRS	1975 1982 1985 1987 2012
244	5	54	$[0.7650, 0.7657]$	DRS	1967 1989 2000 2004 2014
245	5	54	$[1.3055, 1.3066]$	IRS	1958 1961 1977 1995 2005
246	6	53	$[1.0364, 1.0415]$	IRS	1960 1981 1982 1993 1998 2016
247	8	51	$[1.2139, 1.2144]$	IRS	1962 1968 1974 1982 1990 1993 1994 1999
248	5	54	$[1.0894, 1.0906]$	IRS	1963 1971 1982 2015 2016
249	7	52	$[1.1916, 1.1970]$	IRS	1962 1983 1988 1991 1995 2011 2014
250	5	54	$[0.9424, 0.9428]$	DRS	1959 1970 1990 1998 2003
251	5	54	$[1.7313, 1.7323]$	IRS	1980 1990 1991 1999 2003
252	5	54	$[1.2224, 1.2278]$	IRS	1961 1984 1985 1994 2002
253	6	53	$[1.1586, 1.1590]$	IRS	1961 1963 1967 1972 1997 2010
254	5	54	$[1.1212, 1.1247]$	IRS	1958 1959 1977 2001 2005
255	5	54	$[1.1500, 1.1521]$	IRS	1981 1982 1995 2000 2006
256	5	54	$[0.7488, 0.7500]$	DRS	1980 1994 1995 1997 2013
257	5	54	$[1.0751, 1.0803]$	IRS	1958 1959 1991 1996 2012
258	5	54	$[0.8700, 0.8700]$	DRS	1982 1983 1990 1998 2009
259	5	54	$[0.9189, 0.9223]$	DRS	1975 1980 1985 2004 2006
260	5	54	$[1.2641, 1.2684]$	IRS	1958 1960 1965 1968 1997
261	6	53	$[1.3641, 1.3770]$	IRS	1961 1963 1972 1987 2000 2003
262	5	54	$[1.0719, 1.0723]$	IRS	1960 1977 1990 1993 2003
263	8	51	$[2.0475, 2.0644]$	IRS	1969 1980 1982 1986 1988 1992 1998 2001
264	9	50	$[1.7303, 1.7483]$	IRS	1960 1977 1982 1983 1985 1993 1994 1997 2000
265	5	54	$[1.4521, 1.4714]$	IRS	1962 1971 1981 1982 1984
266	7	52	$[1.5619, 1.5678]$	IRS	1960 1965 1968 1989 1994 2004 2008
267	6	53	$[1.0401, 1.0405]$	IRS	1982 1984 1985 1988 1995 2016
268	7	52	$[1.0877, 1.0932]$	IRS	1979 1984 1989 1993 1995 2012 2016
269	5	54	$[2.9583, 3.2705]$	IRS	1964 1979 1985 1994 2006
270	8	51	$[1.3201, 1.3394]$	IRS	1960 1961 1979 1982 1988 1994 1997 2005
271	6	53	$[4.5218, 4.5747]$	IRS	1984 1988 1990 1994 1997 2001
272	7	52	$[1.4543, 1.4665]$	IRS	1961 1974 1978 1983 1990 1993 2010
273	6	53	$[1.4760, 1.5126]$	IRS	1961 1963 1970 2000 2010 2015
274	7	52	$[1.0610, 1.0707]$	IRS	1959 1961 1971 1973 1993 1996 2016
275	7	52	$[1.1215, 1.1227]$	IRS	1971 1984 1986 1987 1990 1999 2005
276	7	52	$[1.0437, 1.0447]$	IRS	1964 1975 1977 1989 1992 1997 2000
277	6	53	$[1.1364, 1.1674]$	IRS	1980 1989 1991 2002 2004 2010
278	6	53	$[1.2065, 1.2086]$	IRS	1959 1962 1973 1995 1996 2012
279	6	53	$[0.7732, 0.7926]$	DRS	1978 1984 1986 1999 2000 2007
280	6	53	$[0.9487, 0.9489]$	DRS	1977 1979 1983 1995 1997 2012
281	7	52	$[1.1857, 1.1963]$	IRS	1964 1972 1983 1984 1992 2009 2016
282	7	52	$[1.3075, 1.3079]$	IRS	1973 1976 1993 1998 2004 2013 2014
283	6	53	$[0.9261, 0.9273]$	DRS	1962 1969 1979 1997 2006 2014
284	6	53	$[1.4312, 1.4415]$	IRS	1959 1963 1973 1996 2005 2016
285	5	54	$[1.7831, 1.8118]$	IRS	1968 1972 1980 1982 1990
286	4	55	$[2.1512, 2.1582]$	IRS	1969 1984 1995 2003
287	5	54	$[0.7686, 0.7760]$	DRS	1970 1979 1991 1995 2014
288	5	54	$[1.0844, 1.0890]$	IRS	1958 1964 1976 1983 1998

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Table C.3: Largest feasible year subset by industry, 1958–2016 (continued)

Industry	$\lvert T_i^* \rvert$	Removed	$\mathcal{K}(S_i(T_i^*))$	Regime	Kept years T_i^*
289	5	54	$[1.2674, 1.2688]$	IRS	1965 1970 1985 1990 2000
290	4	55	$[1.3389, 1.3429]$	IRS	1959 1960 1991 2003
291	4	55	$[4.0606, 4.1365]$	IRS	1965 1984 1995 1997
292	4	55	$[1.6426, 1.6991]$	IRS	1958 1974 1996 1999
293	4	55	$[4.3341, 4.3870]$	IRS	1976 1989 2002 2004
294	4	55	$[1.8818, 1.8977]$	IRS	1980 1993 1996 2003
295	4	55	$[3.1668, 3.2276]$	IRS	1958 1977 1978 1996
296	6	53	$[2.1313, 2.1322]$	IRS	1958 1961 1978 1979 1989 2000
297	5	54	$[1.0640, 1.0644]$	IRS	1969 1986 1997 2002 2016
298	6	53	$[1.3627, 1.3653]$	IRS	1958 1963 1965 1982 1992 1993
299	7	52	$[1.1356, 1.1395]$	IRS	1960 1966 1973 1979 1986 1992 1995
300	5	54	$[0.8735, 0.8741]$	DRS	1989 1990 2004 2013 2016
301	5	54	$[1.6570, 1.6591]$	IRS	1958 1976 1986 1995 2006
302	4	55	$[1.4954, 1.4984]$	IRS	1960 1990 1999 2000
303	4	55	$[1.3170, 1.3227]$	IRS	1961 1990 1994 2000
304	5	54	$[1.5201, 1.5473]$	IRS	1962 1982 1990 1992 2000
305	6	53	$[0.7364, 0.7368]$	DRS	1958 1960 1991 1993 1995 2003
306	6	53	$[1.2865, 1.2877]$	IRS	1961 1962 1963 1988 1989 2001
307	8	51	$[1.3673, 1.3687]$	IRS	1963 1965 1978 1985 1993 1995 2009 2012
308	6	53	$[1.2702, 1.2770]$	IRS	1958 1962 1972 1994 1998 2014
309	6	53	$[1.2146, 1.2160]$	IRS	1962 1966 1972 1996 1998 2006
310	5	54	$[1.5197, 1.5404]$	IRS	1959 1962 1974 1987 1992
311	5	54	$[1.2129, 1.2166]$	IRS	1958 1960 1964 1977 1982
312	5	54	$[1.2074, 1.2077]$	IRS	1958 1964 1971 2003 2009
313	7	52	$[1.0605, 1.0655]$	IRS	1958 1960 1979 1990 1996 2000 2002
314	5	54	$[1.0913, 1.0933]$	IRS	1981 1989 1993 1998 2015
315	5	54	$[0.9662, 0.9747]$	DRS	1981 1984 1988 1998 2002
316	6	53	$[1.2243, 1.2259]$	IRS	1958 1963 1970 1991 2003 2004
317	4	55	$[1.2516, 1.2615]$	IRS	1982 1983 1998 2002
318	5	54	$[1.0432, 1.0442]$	IRS	1984 1992 1997 2000 2003
319	5	54	$[1.1308, 1.1329]$	IRS	1988 1992 1995 1997 2003
320	5	54	$[1.1182, 1.1192]$	IRS	1962 1970 1995 2010 2016
321	5	54	$[1.4822, 1.4876]$	IRS	1962 1991 1995 1998 1999
322	5	54	$[1.1407, 1.1424]$	IRS	1977 1982 1983 1997 2004
323	6	53	$[0.9684, 0.9751]$	DRS	1964 1965 1982 2010 2013 2015
324	6	53	$[1.4367, 1.4379]$	IRS	1970 1990 1993 1994 1999 2001
325	5	54	$[2.0792, 2.1265]$	IRS	1958 1980 2001 2014 2016
326	6	53	$[1.1484, 1.1599]$	IRS	1971 1979 1991 1995 2006 2010
327	4	55	$[0.6090, 0.6120]$	DRS	1988 1997 2014 2016
328	5	54	$[0.7303, 0.7309]$	DRS	1963 1966 1970 2014 2015
329	5	54	$[1.0085, 1.0152]$	IRS	1982 1998 2006 2008 2009
330	5	54	$[1.5250, 1.5406]$	IRS	1959 1960 1963 1984 2007
331	6	53	$[0.9657, 0.9713]$	DRS	1967 1970 2000 2002 2014 2015
332	6	53	$[1.4395, 1.4418]$	IRS	1961 1963 1964 1968 1990 2007
333	5	54	$[0.8258, 0.8260]$	DRS	1975 1980 1987 1993 2009
334	5	54	$[1.0698, 1.0701]$	IRS	1958 1962 1991 1992 2008
335	6	53	$[1.1111, 1.1114]$	IRS	1980 1984 1991 1993 1999 2009
336	6	53	$[1.5941, 1.5954]$	IRS	1966 1978 1980 1991 1993 2009

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Table C.3: Largest feasible year subset by industry, 1958–2016 (continued)

Industry	$\lvert T_i^* \rvert$	Removed	$\mathcal{K}(S_i(T_i^*))$	Regime	Kept years T_i^*
337	6	53	$[1.1669, 1.1679]$	IRS	1958 1960 1963 1969 1991 2000
338	4	55	$[1.2952, 1.2981]$	IRS	1991 1992 1998 2001
339	6	53	$[1.3748, 1.3786]$	IRS	1960 1962 1963 1998 1999 2009
340	5	54	$[1.3294, 1.3319]$	IRS	1964 1968 1973 2011 2016
341	5	54	$[0.8405, 0.8412]$	DRS	1977 1980 1997 2001 2002
342	5	54	$[1.0110, 1.0188]$	IRS	1969 1981 1997 2003 2010
343	6	53	$[1.1843, 1.1868]$	IRS	1959 1961 1981 1983 2000 2011
344	5	54	$[1.3987, 1.4002]$	IRS	1960 1962 1994 2005 2006
345	5	54	$[2.4625, 2.4995]$	IRS	1959 1961 1972 1998 2009
346	4	55	$[2.4442, 2.4677]$	IRS	1961 1962 1966 1972
347	6	53	$[1.3884, 1.3934]$	IRS	1977 1981 1982 2000 2006 2016
348	5	54	$[1.2255, 1.2267]$	IRS	1962 1967 1969 1990 2015
349	5	54	$[0.5927, 0.5931]$	DRS	1979 1980 1994 1999 2010
350	6	53	$[1.6375, 1.6723]$	IRS	1985 1990 1994 1998 2003 2014
351	5	54	$[3.4572, 3.4816]$	IRS	1973 1982 1985 1996 2004
352	4	55	$[1.6432, 1.6499]$	IRS	1958 1981 1987 2008
353	5	54	$[0.6603, 0.6619]$	DRS	1980 1988 1993 2003 2006
354	4	55	$[2.0464, 2.0787]$	IRS	1959 1974 1995 1998
355	5	54	$[1.0483, 1.0527]$	IRS	1958 1960 1961 1985 2006
356	4	55	$[0.8997, 0.9026]$	DRS	1958 1960 1995 2009
357	6	53	$[0.9537, 0.9540]$	DRS	1980 1991 1992 1994 1997 2014
358	5	54	$[0.5709, 0.5721]$	DRS	1982 1988 1992 1999 2015
359	4	55	$[0.9131, 0.9204]$	DRS	1990 1998 2000 2008
360	4	55	$[1.5050, 1.5094]$	IRS	1983 1993 2007 2012
361	5	54	$[1.5081, 1.5136]$	IRS	1958 1968 1974 1992 2008

Notes: Industry is the industry identifier used throughout the paper; $|T_i^*|$ is the size of the largest subset of years for which $\mathcal{K}(S_i(T)) \neq \varnothing$ (Section 4.4); Removed is $|T_i| - |T_i^*|$, the number of years dropped to achieve this; $\mathcal{K}(S_i(T_i^*)) = [\underline{k}, \overline{k}]$ is the identified set over the retained years; Regime is IRS/DRS required if the interval lies entirely above/below one, or CRS admissible if $1 \in \mathcal{K}(S_i(T_i^*))$; Kept years T_i^* lists the retained years. If multiple year subsets attain the same maximum cardinality, one such subset is reported. Capital is excluded from the priced input vector entirely in this four-input specification, $x_{it} = (L_{it}, N_{it}, M_{it}, E_{it})$ (Section 4.7).

Table 6: Cross-industry heterogeneity in consistency with a stable homogeneous technology

	(1) $e^*(S_i)$	(2) $L_i^{\max}$	(3) $\log k_i$
Capital share	-1.301^{***}	-1.858	1.923
	(0.330)	(1.168)	(1.650)
Materials share	-0.443^{***}	-0.551^{**}	-1.017^{**}
	(0.074)	(0.249)	(0.398)
Energy share	-0.159	2.476^{***}	-1.638
	(0.270)	(0.880)	(1.875)
Output growth	-0.153	3.467^{***}	1.642
	(0.318)	(1.069)	(1.598)
Output-growth volatility	-0.676^{***}	-0.474	-0.746
	(0.157)	(0.517)	(1.256)
Capital deepening	-2.912^{***}	1.185	-4.340
	(0.570)	(1.833)	(3.059)
log size	0.026^{***}	0.001	0.113^{**}
	(0.006)	(0.026)	(0.049)
Constant	1.021^{***}	2.584^{***}	0.015
	(0.063)	(0.250)	(0.464)
Observations	361	361	361
R^2	0.327	0.066	0.053

Notes: OLS with heteroskedasticity-robust standard errors in parentheses. ***, **, * denote significance at 1%, 5%, 10%. Column (1): cost-efficiency index over the full 1958–2016 panel. Column (2): longest consecutive rationalizable window $L_i^{\max}$. Column (3): log of the geometric midpoint of $\mathcal{K}$ over one selected longest rationalizable window when more than one window attains the maximum length; this column is therefore descriptive and need not be selection-invariant. Regressors are industry means or mean annual log growth rates over 1958–2016: factor shares in total measured cost; output growth and its standard deviation; capital deepening is the mean log growth of K/L; size is the log of mean total measured cost. Baseline specification, $r = 8\%$.

Table 7: Sensitivity to the capital-price construction and the input set

	Nonempty $\mathcal{K}(S_i)$		Consecutive windows passing, by length L			
Specification	1958–2016	2014–2016	$L = 2$	$L = 3$	$L = 4$	$L \geq 5$
User cost, $r = 6\%$	0	2	12,999	119	1	0
User cost, $r = 8\%$ (baseline)	0	1	12,964	106	0	0
User cost, $r = 10\%$	0	1	12,898	122	2	0
Four inputs (capital excluded)	0	2	13,136	116	1	0
Windows tested			20,938	20,577	20,216	$\leq$ 19,855

Notes: The four-input specification uses $x_{it} = (L_{it}, N_{it}, M_{it}, E_{it})$, treating capital as quasi-fixed and excluding it from the priced input vector entirely, so it does not depend on how capital is priced. The $L \geq 5$ column reports the total number of passing windows over all lengths $L \in \{5, 6, 8, 10, 15, 20\}$ tested: it is exactly zero in every specification. At $r = 6\%$ the two nonempty short-panel industries are 45 and 78; industry 45's identified set is unbounded above. Industry 78 is nonempty at every r. Under the four-input specification the mean m^* is 4.97 against 5.22 in the baseline, the median, minimum, and maximum are unchanged at 5, 3 and 9, and mean d_N rises from 0.912 to 0.916.

Table C.1: Speed of breakdown beyond the maximum feasible subset, the three industries with $m^* = 9$

Industry	$\mathcal{K}(T_i^*)$	$g(T_i^*)$	$\lvert T\rvert$ at $g \geq 0.1$	$\lvert T\rvert$ at $g \geq 1$	$\lvert T\rvert$ at $g \geq 5$	g at $\lvert T\rvert = 59$
264	$[1.7717, 1.7731]$	0	12	18	41	1174.6
270	$[1.1651, 1.1750]$	0	11	28	49	2139.5
307	$[1.2755, 1.2766]$	0	12	23	48	111.9

Notes: $g(T_i^*) = 0$ by construction at the departure point. Columns 4–6 report the number of years retained along the greedy path at which g first reaches 0.1, 1, and 5. Baseline specification, $r = 8\%$. Because maximizers are not unique (Section 4.4), the path depends on which maximizer is the departure point; the endpoint does not.